String Theory

A general overview & current “hot” topics

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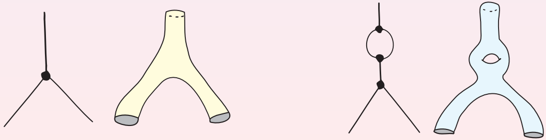
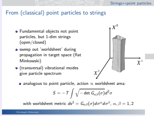
- 4d model building
- Non-perturbative aspects
- *Optional:*
  Vafa’s F-theory GUT model building

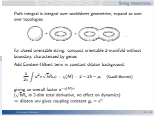
Würzburg — January 8th, 2009



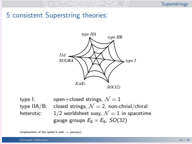
Last time (→ talk by Christoph Uhlemann)

- Bosonic string:
  - Nambu-Goto and Polyakov action
  - Mode expansion and Virasoro algebra
  - Interactions and scattering

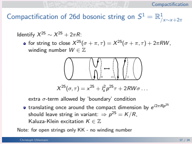




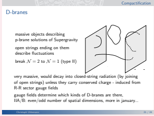
- Superstrings:
  - Worldsheet SUSY, spacetime fermions
  - Type-I and type-II superstring spectrum
  - Spacetime SUSY

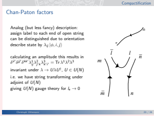


- Toroidal compactification:
  - Circle compactification of the bosonic string
  - 2-Torus compactification and modular invariance
  - Winding modes and T-duality



- D-branes
- Chan-Paton factors





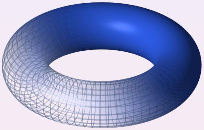
Part I

4d model building

- Calabi-Yau compactification
- Orbifolds
- Flux compactification and moduli stabilization
- D-branes and orientifolds

SUSY conditions I: Spinor splitting

- Compactifying the 10d $\mathcal{N} = 2$ type-II superstring on a 6-torus yields (maximal) 4d $\mathcal{N} = 8$ supersymmetry in the effective theory.
  ➙ no chiral 4d fermions in toroidal compactifications



New approach: Investigate the effective 10d $\mathcal{N} = 2$ type-II SUGRA and find conditions for 4d minimal supersymmetry.

1. Assume a product space-time $\mathcal{M}_{10} = \mathbb{R}^{1,3} \times K^6$, where K^6 is some compact inner space.
   ➙ 4d Poincaré-invariance: All the fermionic fields must be trivial!

SUSY conditions II: No-flux restrictions

SUSY variations without R-R fluxes

bosons:	All vanish due to the trivial fermionic fields (parity)
gravitino:	$\delta_\epsilon \chi_\mu = \nabla_\mu \epsilon - \frac{1}{4} H_{\mu\nu\rho} \gamma^{[\nu} \gamma^{\rho]} \epsilon + (\text{fermions})^2$
dilatino:	$\delta_\epsilon \varphi = (\gamma^\mu \nabla_\mu \Phi)\, \epsilon + \frac{1}{24} H_{\mu\nu\rho} \gamma^{[\mu} \gamma^\nu \gamma^{\rho]} \epsilon + (\text{fermions})^2$

One imposes additional simplifications in order to make the SUSY variations of the fermionic fields vanish:

2. Consider a constant dilaton field $\Phi = \Phi_0$.
3. Consider a vanishing NS-NS background flux, i.e. $H_3 = 0$.
   ➙ Dilatino equation automatically satisfied.

Remaining no-flux SUSY variations

gravitino: $\delta_\epsilon \chi_\mu = \nabla_\mu \epsilon$

SUSY conditions III: Geometric implications

- On $\mathbb{R}^{1,3} \times K^6$ the equation $\nabla_\mu \epsilon = 0$ splits into two conditions:
  1. Constant spinor on $\mathbb{R}^{1,3}$ ➙ trivial condition.
  2. Covariantly constant, nowhere vanishing spinor on compact space K^6
     ➙ severe constraints on the metric and topology.

Theorem of Wang

K^6 is Ricci-flat and Kähler.

Berger's holonomy classification

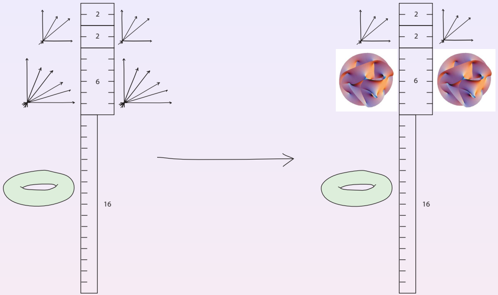
K^6 has $\mathrm{SU}(3)$-holonomy.

[*Holonomy:* What could possibly happen to a tangent vector, when it is parallel transported along a closed loop?


]

- A 6d $\mathrm{SU}(3)$-holonomy manifold is just a Calabi-Yau manifold.
  ➙ The internal space K^6 must be a Calabi-Yau 3-fold.

Calabi-Yau compactification I: Basic setup



[In case of the heterotic string, many more technical subleties have to be considered.
➙ Embedding of the spin connection in the gauge degrees of freedom]

In string theory model building only the massless spectrum is usually considered. The primary interest is in the associated effective SUGRA theory obtained by compactification:

- Heterotic string theory on CY 3-fold ➙ 4d $\mathcal{N} = 1$ het. SUGRA
- Type-II string theory on CY 3-fold ➙ 4d $\mathcal{N} = 2$ type-II SUGRA
  ➙ Need to break SUSY further down do $\mathcal{N} = 1$.

Calabi-Yau compactification II: Effective field content

- Problem: Non-trivial CY 3-fold metrics are not explicitly known
  ➙ only access to topology of the compact space K^6.

Type-IIA on CY 3-fold

$h^{1,1}$ Abelian vector multiplets
$h^{2,1}+1$ hypermultiplets

Type-IIB on CY 3-fold

$h^{2,1}$ Abelian vector multiplets
$h^{1,1}+1$ hypermultiplets

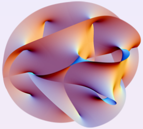
[*Hodge numbers:* $h^{p,q} := \dim_{\mathbb{R}} H^{p,q}_{\bar{\partial}}(K^6)$ are the dimensions of the Dolbeault cohomology groups (c.f. Betti numbers).]

- The massless scalars in those $\mathcal{N}=2$ multiplets of the effective theory parameterize the geometric moduli space, which is a product space:

$$\mathcal{M}_{\mathrm{geom}}(K^6) = \mathcal{M}^{1,1}(K^6) \times \mathcal{M}^{2,1}(K^6).$$

The expectation values of the massless scalars are determined by the values of the geometric moduli.

Calabi-Yau compactification III: Massless scalars & moduli



Number of massless scalars in effective theory

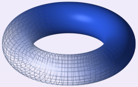
- type-IIA ➙ $2h^{1,1} + 4(h^{2,1} + 1)$ scalars
- type-IIB ➙ $2h^{2,1} + 4(h^{1,1} + 1)$ scalars

[*Scalars:* An $\mathcal{N} = 2$ hypermultiplet has 4 real scalars, an Abelian vector multiplet has 2 real scalars]

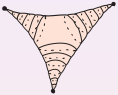
- Changing the value of the massless scalars does not require any energy, therefore they may take any value.
- Such unconstrained massless scalars give rise to an additional 5th interaction unobserved in nature.
  ➙ The scalars have to disappear / be fixed.
- But unfortunately in the effective type-II supergravity there is no potential for the massless scalars...
  ➙ Moduli stabilization problem

Compactifications: A *preliminary* overview

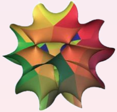
- Toroidal compactification:
  - Very simple to compute using periodic coordinates.
  - No supersymmetry breaking ($\mathcal{N} = 8$ for type-II)
    → phenomenologically ruled out



- Orbifold compactification:
  - Computations quite similar to toroidal compactifications
  - Singular space-time
  - Breaks supersymmetry to minimal levels ($\mathcal{N} = 2$ for type-II)
    → need further SUSY breaking



- Calabi-Yau compactification:
  - Breaks supersymmetry to minimal levels ($\mathcal{N} = 2$ for type-II)
  - Only access to topological data of the internal space
  - Introduces lots of massless scalars (geometric moduli)
    → need moduli stabilization and further SUSY breaking



→ need further ingredients: Fluxes & D_p-branes & orientifolds

Flux compactification I: Background RR-fluxes

R-R p-forms in type-IIA

opp. chirality: ${}_{\mathrm{L}}|+\rangle \otimes |-\rangle_{\mathrm{R}}$
➞ C_1, C_3, C_5, C_7, C_9

R-R p-forms in type-IIB

same chirality: ${}_{\mathrm{L}}|+\rangle \otimes |+\rangle_{\mathrm{R}}$
➞ C_0, C_2, C_4, C_6, C_8, C_{10}

- NS-NS flux: $H_3 := \mathrm{d}B_2$ *(field-strenth of the NS-NS 2-form)*
- R-R fluxes: $F := \mathrm{d}C - H_3 \wedge C$ *(formal sum of all even/odd fluxes)*

$$F_0 = - \star F_{10} \qquad F_1 = \mathrm{d}C_0$$
$$F_2 = \mathrm{d}C_1 \qquad F_3 = \mathrm{d}C_2 - H_3 C_0$$
$$F_4 = \mathrm{d}C_3 - H_3 \wedge C_1 \qquad F_5 = \mathrm{d}C_4 - H_3 \wedge C_2$$

- Hodge self-duality constraint: $F_n = (-1)^{\lfloor \frac{n}{2} \rfloor} \star F_{10-n}$
- Bianchi identities: $\mathrm{d}H = 0, \qquad \mathrm{d}F - H \wedge F = 0$
- Basic idea: In the presence of background fluxes, changing the value of the massless scalars requires energy.
  ➞ Massless scalars obtain dynamically stabilized values

Flux compactification II: Geometric implications

- In order to break the supersymmetry to minimal levels as before, one has to consider the SUSY variations of the fields including the R-R fluxes.

Relevant SUSY variations to first order in ϵ including R-R fluxes

bosons:	Vanish due to trivial fermionic fields
gravitino:	$\delta_\epsilon \chi_\mu = \nabla_\mu \epsilon + \frac{1}{4} \not{H}_\mu \mathcal{P} \epsilon + \frac{1}{16} \mathrm{e}^\Phi \sum_n \not{F}_n \gamma_\mu \mathcal{P}_n \epsilon$
dilatino:	$\delta_\epsilon \varphi = \left(\not{\partial} \phi + \frac{1}{2} \not{H} \mathcal{P} \right) \epsilon + \frac{1}{8} \mathrm{e}^\Phi \sum_n (-1)^n (5-n) \not{F}_n \mathcal{P}_n \epsilon$

- Actually solving the above conditions requires much further techniques:
  - Generalized complex geometry ➞ Generalized Calabi-Yau manifolds

This deals with one of the two issues mentioned.
➞ Still need to further break SUSY to $\mathcal{N} = 1$.

D-branes I: Minimal coupling

- Open strings can have either von-Neumann or Dirichlet boundary conditions. In case of the latter, their space-time position is fixed to a submanifold. ➙ D_p-brane.
- D-branes are minimally charged under type-II R-R p-form fields.

Electric coupling

The natural coupling of a $(p+1)$-form to a p-brane is given by

$$Q_p \int_{\mathcal{W}_p} A_{p+1}$$

➙ electric p-brane with charge Q_p

Magnetic coupling

The magnetic dual to the field strength $F_{p+2} := \mathrm{d}A_{p+1}$ is defined by the Hodge dual $\tilde{F} := \star F$, i.e.

$$\mathrm{d}\tilde{A}_{D-p-3} = \tilde{F}_{D-p-2} = \star F_{p+2} = \star \mathrm{d}A_{p+1}.$$

➙ magnetic $(D-p-4)$-brane with charge $\tilde{Q}_{D-p-4}$.

D-branes II: Worldvolume actions

- Open string excitations induce a $\mathrm{U}(1)$-gauge theory on the $(p+1)$-dimensional D_p-brane-worldvolume $\mathcal{W}_p$, which couples to all the lower R-R fields $C_{p+1}, C_{p-1}, C_{p-3}, \ldots$

Worldvolume actions of D-branes

Dirac-Born-Infeld: $S_{\mathrm{DBI}} = -T_p \int_{\mathcal{W}_p} e^{-\phi} \sqrt{-\det(g + B_2 + 2\pi\alpha' F_2)}$

Chern-Simons: $S_{\mathrm{CS}} = \frac{T_p}{2} \int_{\mathcal{W}_p} \underbrace{C \wedge \mathrm{ch}(F_2)}_{\text{coupling}} \wedge \underbrace{\sqrt{\frac{\widehat{A}(R_{\mathrm{T}\mathcal{W}_p})}{\widehat{A}(R_{\mathrm{N}\mathcal{W}_p})}}}_{\text{required for gravitational anomaly cancellation}} \Bigg|_{\text{(p+1)-form}}$

[T_p tension of D_p-brane
F_2 worldvolume $\mathrm{U}(1)$ gauge field-strength]

$$\left[\textit{Example:} \quad C \wedge \mathrm{ch}(F_2)\big|_{\text{8-form}} = C_8 + C_6 \wedge \mathrm{Tr}\, F_2 + C_4 \wedge \mathrm{Tr}(F_2 \wedge F_2) + \ldots\right]$$

D-branes III: Multiple D-branes

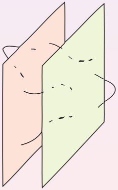
D_p-branes in type-IIA

D0 *(point particle)*,
D2, D4, D6, D8

D_p-branes in type-IIB

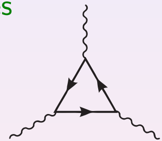
D(-1) *(D-instanton)*, D1 *(D-string)*,
D3, D5, D7, D9 *(space-time)*

- Open strings can stretch between several D-branes.
  - A collection of N coinciding D_p-branes is called a stack
    ➙ the stack's worldvolume gauge group enhances to $\mathrm{U}(N)$.
  - The branes of a stack can dynamically move apart
    ➙ gauge symmetry breaking $\mathrm{U}(N) \mapsto \mathrm{U}(N_1) \times \mathrm{U}(N_2)$.
  - Intersecting stacks of branes can be used to generate chiral fermions. The states resulting from the intersection transform in the bi-fundamental representation of the stacks' gauge groups, i.e. $(\square_a, \square_b)$ or $(\square_a, \bar{\square}_b)$. ➙ bi-fundamental (chiral) matter



D-branes IV: Global issues

- In a compactification scenario $\mathbb{R}^{1,3} \times K^6$ a D_p-brane occupies a $(p-3)$-dimensional submanifold of K^6. In the compact directions the total charge and tension must cancel. ➙ Chiral anomalies



> *Recall:* According to the Gauß theorem from classical electrodynamics, the force / field-strength in a compact volume is determined by the total charge content.

➙ Tadpole and consistency conditions.

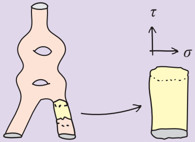
- Introduce anti-D_p-branes: Same geometric properties as ordinary D-branes, but negative charge.

  ➙ Charge cancellation using D-branes and anti-D-branes

Orientifolds I: Worldsheet parity operator

- On the type-II closed superstring, the left- and right-moving are independant of each other except for the level matching ("equal mass") condition. ➙ $\mathcal{N} = 2$ SUSY
- Consider worldsheet orientation-reversing actions.

Worldsheet parity operator Ω_{P}



Action of the worldsheet parity operator on the worldsheet: $\Omega_{\mathrm{P}} : \sigma \mapsto 2\pi - \sigma$

➙ reverses worldsheet orientation

➙ Consider the coset space $\mathcal{M}_{10}/(\Omega_{\mathrm{P}} P)$.

- The action of P on the space-time is usually generated by an involution ξ, i.e. a mapping $\xi : \mathcal{M}_{10} \longrightarrow \mathcal{M}_{10}$ where $\xi^2 = \mathrm{Id}$.

Orientifolding in type-IIA

action: $\underbrace{(-1)^{F_{\mathrm{L}}}\Omega_{\mathrm{P}}\xi}_{\text{O6}}$

$\left[\begin{array}{l}\text{holomorphic volume: } \xi^*\Omega = \bar{\Omega} \\ \text{complex structure: } \xi^* J = -J\end{array}\right]$

Orientifolding in type-IIB

action: $\underbrace{(-1)^{F_{\mathrm{L}}}\Omega_{\mathrm{P}}\xi}_{\text{O5/O9}}$ or $\underbrace{\Omega_{\mathrm{P}}\xi}_{\text{O3/O7}}$

$\left[\begin{array}{l}\text{holomorphic volume: } \xi^*\Omega = \pm\Omega \\ \text{complex structure: } \xi^* J = J\end{array}\right]$

- The fixpoint set of the orientifold involution ξ of the space-time is called an orientifold plane.
- Supersymmetry considerations restrict the possible types of O-planes.

Charge and tension of D-branes and O-planes

	D	$\bar{\text{D}}$	O_+	O_-
charge	$+$	$-$	$-$	$+$
tension	$+$	$+$	$-$	$+$

➙ Tension cancellation using O-planes.

"Bottom up" model building

- *Example:* Orientifolding the *(oriented, closed)* type-IIB superstring yields the *(unoriented, open/closed)* type-I superstring.

Calabi-Yau orientifold model building 1-0-1

1. Calabi-Yau geometry ensures 4d $\mathcal{N} = 2$ supersymmetry.
2. Adding D$_p$-branes and O-planes breaks SUSY to 4d $\mathcal{N} = 1$ and provides the means of $\mathrm{U}(n)$, $\mathrm{SO}(n)$ or $\mathrm{Sp}(n)$ gauge theory.
3. Background fluxes can be used to stabilize the moduli.

➜ IIA and IIB model building with 4d $\mathcal{N} = 1$ eff. theories

- In principle, we now have all the neccessary tools for perturbative "bottom-up" 4d model building.
  - ➜ What about the non-perturbative side?

Part II

Non-perturbative aspects

- Duality web: T-, S- and U-duality
- M-theory
- F-theory as non-perturbative type-IIB theory

Five consistent 10d perturbative superstring theories

I	open and closed strings in bulk, $\mathrm{SO}(32)$ gauge symmetry
IIA	closed strings in bulk, open strings ending on D-branes, non-chiral
IIB	closed strings in bulk, open strings ending on D-branes, chiral
HE	only closed strings in bulk, $\mathrm{E}_8 \times \mathrm{E}_8$ gauge symmetry
HO	only closed strings in bulk, $\mathrm{SO}(32)$ gauge symmetry

The five superstring theories are related via numerous dualities:

1. T-duality: The radius R of a compact direction with the topology of a circle is mapped to $\frac{1}{R}$ and "wrapped" string states will be exchanged with high-momentum string states.

$$R \leftrightarrow \frac{1}{R} \qquad \overbrace{n \leftrightarrow w}^{\text{exchange of state types}}$$

Examples of T-duality:

- Type-IIA on radius R is T-dual to type-IIB on radius $\frac{1}{R}$
- Heterotic $\mathrm{E}_8 \times \mathrm{E}_8$-string is T-dual to the heterotic $\mathrm{SO}(32)$-string.

➙ T-duality relates the small to the large dimensions.

2. S-duality: Inverts the string coupling, i.e. $g_s \leftrightarrow \frac{1}{g_s}$.

   [The string coupling depends on the dilaton: $g_s := e^{-\Phi} \iff \Phi \mapsto -\Phi$.]

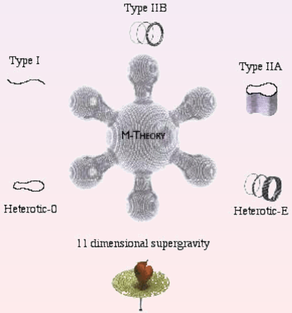
Examples of S-duality:

- Type-I superstring is S-dual to the heterotic $\mathrm{SO}(32)$-string
- Type-IIA superstring is S-dual to the heterotic $\mathrm{E}_8 \times \mathrm{E}_8$-string
- The type-IIB superstring ist self-dual under S-duality!

➙ S-duality relates the weak and strong coupling regime.

3. U-duality: combination of S- and T-duality in the context of M-theory.

- Witten (1995): "If all the perturbative superstring theories are dual to each other, there should be a common non-perturbative origin!"
  → M-theory → sparked the "2nd superstring revolution"
- Basic idea: The 5 superstring theories should be certain limiting cases of M-theory.
- Main problem: As yet there is no microscopic formulation of M-theory, i.e. there's no such thing like a "M-theory lagrangian".



- Different approaches:
  1. Formulation as the non-perturbative origin of type-IIA superstring theory
  2. "Matrix model" formulation as a membrane theory
  3. UV-completion of the 11d $\mathcal{N} = 1$ SUGRA

M-Theory II: Type-IIA at strong coupling

- Important observation: The effective 10d type-IIA SUGRA theory is equivalent to the dimensional reduction of 11d SUGRA.

Lowest order 11d $\mathcal{N} = 1$ and type-IIA supergravity (bosonic part)

dimensional reduction: ignore field's dependency on 11th coord.; $G_4 := \mathrm{d}\hat{C}_3$

$$S^{\mathrm{bos}}_{\mathrm{11d}} = \frac{1}{2\kappa^2} \int \mathrm{d}^{11}x \sqrt{\tilde{g}} \left[\tilde{R} - \frac{1}{48} |G_4|^2 \right] + \frac{1}{4\kappa^2} \int \hat{C}_3 \wedge G_4 \wedge G_4$$

$$S^{\mathrm{bos}}_{\mathrm{IIA}} = \frac{1}{2\kappa^2} \int \mathrm{d}^{10}x \sqrt{g} \left[\mathrm{e}^{-2\Phi} \left(R + 4(\nabla\Phi)^2 - \frac{|H_3|^2}{12} \right) \underbrace{- \frac{|F_4|^2}{48} - \frac{|F_2|^2}{2}}_{\text{R-R kinetic terms do not couple to the dilaton}} \right]$$

(String frame)
$$+ \frac{1}{4\kappa^2} \int B_2 \wedge \mathrm{d}C_3 \wedge \mathrm{d}C_3$$

- In the dimensional reduction, the diagonal component $\tilde{g}_{11,11}$ of the 11d metric becomes the string coupling $g_s = \mathrm{e}^{-\Phi}$
  - ➙ Type-IIA apparently "grows" a 11th dimension for $g_s \to \infty$

M-Theory III: M2-branes and matrix theory

- The 11d 3-form field $\hat{C}_3$ naturally couples to 2-branes ➙ M2-branes
- M2-branes are supposedly the "fundamental" objects of M-theory, just like strings are the fundamental objects of (perturbative) string theory
- In matrix theory one aims to find a direct quantization of such "membranes" in order to find a direct description of M-theory
  - Quantization conditions seem to be too restrictive
  - Classification of 3-manifolds is vastly more difficult compared to the classification of 2-manifolds (string worldsheet topologies)

    [For example, the Poincaré conjecture (which states that every simply connected, compact 3-manifold (without boundary) is homeomorphic to the 3-sphere) was first solved in 2003 by Grigori Perelman (and led to his - rejected! - Fields Medal in 2006)]

➙ Many unsolved problems in finding a proper formulation of M-theory...

F-Theory I: Enhanced S-duality of type-IIB

Lowest order of 10d $\mathcal{N} = 2$ type-IIB supergravity (bosonic part)

$$S_{\mathrm{IIB}}^{\mathrm{bos}} = \frac{1}{2\kappa^2} \int \mathrm{d}^{10}x \sqrt{g} \left[R - \frac{1}{2} \frac{\partial S \, \partial \bar{S}}{(\operatorname{Im} S)^2} - \frac{|G_3|^2}{12} - \frac{|F_5|^2}{240} \right]$$

(Einstein frame)

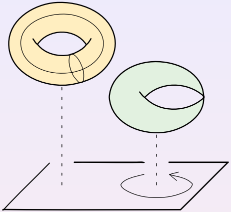
$$+ \frac{1}{2\mathrm{i}} \int C_4 \wedge G_3 \wedge \bar{G}_3 \qquad \text{where} \quad G_3 := \mathrm{i} \frac{F_3 + S H_3}{\sqrt{\operatorname{Im} S}}$$

- In case of type-IIB the S-duality group is enhanced to $\mathrm{SL}(2;\mathbb{R})$
    - Define the (complex) axio-dilaton $S := C_0 + \mathrm{i}e^{-\phi}$ scalar field.
    - Some matrix $\Lambda := \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \mathrm{SL}(2;\mathbb{R})$ acts on the fields

$$S \mapsto \Lambda S := \frac{aS + b}{cS + d} \qquad \begin{pmatrix} B_2 \\ C_2 \end{pmatrix} \mapsto (\Lambda^{-1})^{\mathrm{t}} \begin{pmatrix} B_2 \\ C_2 \end{pmatrix}$$

    - Due to the Dirac quantization condition, the group is reduced to integer coefficents ➙ $\mathrm{SL}(2;\mathbb{Z})$ in quantum theory

F-Theory II: Geometrization of the axio-dilaton



- Vafa (1996): Since $\mathrm{SL}(2;\mathbb{Z})$ is the torus amplitude's modular group, one should interpret the axio-dilaton value at each space-time point to parameterize a 2-torus complex structure and geometry.

- 12d total space $\mathcal{X} \xrightarrow{\pi} \mathcal{M}$ elliptically-fibered over 10d space-time $\mathcal{M}$
- Due to SUSY constraints one takes an elliptically-fibered compact space X_4 with a Calabi-Yau 3-fold base B_3.
  - *Usually:* Elliptically-fibered Calabi-Yau 4-fold $X_4 \xrightarrow{\pi} B_3$
  - *Sometimes:* Elliptically-fibered $\mathrm{Spin}(7)$-holonomy manifold $Y \xrightarrow{\pi} B_3$
- Consider space-time varying dilaton backgrounds with string coupling g_s being weak and strong ➙ F-theory = non-perturbative type-IIB

F-Theory III: 7-branes and singularities

- The R-R scalar C_0 couples magnetically to D7-branes.
- Consider a closed loop γ in the 2d transverse space to the 8d D7-worldvolume. Walking along γ, the value of S changes value!
  - ➙ Monodromy: $S \mapsto S + 1$
  - ➙ Axion-dilaton singularity at the position of the D7-brane!
  - ➙ Singularity in the elliptic fibre!

F-theory model building 1-0-1

1. Take an elliptically-fibered compact space $X_4 \xrightarrow{\pi} B_3$.
   ➙ stage: $\underbrace{\mathcal{X} = \mathbb{R}^{3,1} \times X_4}_{\text{12d total space}} \xrightarrow{\mathrm{pr}_1 \times \pi} \underbrace{\mathcal{M} = \mathbb{R}^{3,1} \times B_3}_{\text{10d space-time}}$
2. 7-branes are located at those points in space-time, where the elliptic fibre degenerates.
3. Non-perturbative type-IIB with space-time varying g_s is described.

(Optional) Part III

Recent Developments

- More on F-theory
- F-theory GUTs with exceptional gauge groups (Vafa et al. '08)

- The degeneration of the elliptic fibre in mathematically understood in terms of Enriques-Kodaire singularities.

Singularities of Kodaire type I_1 ("double point")

Let $\alpha, \beta \in H_1(T^2) \cong \pi_1(T^2) \cong \mathbb{Z} \oplus \mathbb{Z}$ denote the two generators corresponding to the two independant circles on a torus.
➙ torus degeneration: 1-cycle $p\alpha + q\beta$ collapses

- The objects located at such degeneration points are (p, q) 7-branes.
- (p, q) 7-branes can be treated as the image of ordinary D7-branes under appropiate $\mathrm{SL}(2;\mathbb{Z})$-transformations.
  - Ordinary D7-branes are identified as $(1, 0)$ 7-branes
  - O7-planes can be resolved into certain pairs of (p, q) 7-branes
- In the same spirit one defines (p, q)-strings connecting the 7-branes.

Singularities of other Kodaire type: ➙ ADE singularities

- Consider an elliptically-fibered CY 4-fold $X_4 \xrightarrow{\pi} B_3$ and let $S_2 \subset B_3$ denote the singularity locus where the fibre degenerates.
- Let $K_S \xrightarrow{\pi_K} S$ denote the canonical line bundle and $N_S \xrightarrow{\pi_N} S$ the normal line bundle of $S \subset B$.
- The total space of $X|_S \xrightarrow{\pi} S$ looks like an isolated ADE surface singularity fibered over S.

ADE singularities

A_n	$f = y^2 - x^2 - z^{n+1}$	$\mathrm{SU}(n+1)$
D_n	$f = y^2 - x^2 z - z^{n-1}$	$\mathrm{SO}(2n)$
E_6	$f = y^2 - x^3 - z^4$	E_6
E_7	$f = y^2 - x^3 - xz^3$	E_7
E_8	$f = y^2 - x^3 - z^5$	E_8

Let x, y, z be the fibre coordinates (sections) of $K_S^a \oplus K_S^b \oplus N_S \longrightarrow S$.

The subset $\{f(x, y, z) = 0\}$ of this bundle's total space describes $X|_S \xrightarrow{\pi} S$.

F-Theory VI: ADE branes and gauge groups

- The ADE singularity determines the worldvolume gauge group of the corresponding ADE 7-brane.
- Correspondence to the perturbative constructions:
  - $\mathrm{SU}(n+1)$: Realized by stacks of $n+1$ coincident D7-branes.
  - $\mathrm{SO}(2n)$: Realized by D7-branes on top of O7-planes.
  - $\mathrm{E}_6, \mathrm{E}_7, \mathrm{E}_8$: Can be realized by using specific arrangements of (p, q) 7-branes which are connected by appropiate (p, q)-string networks.

  ➙ The F-theory approach naturally yields exceptional gauge groups!
- Given two ADE 7-branes on $S, S' \subset B$ with gauge groups G, G' the gauge group / singularity enhances over the intersection: $G_\Sigma \supset G \times G'$.

Vafa et al. I: Basic ideas

The basic ideas of the work of Vafa et al:

1. Consider only local models, i.e. the elliptically-fibered CY 4-fold is only considered in a small neighbourhood of the singular locus $S \subset B$.
   ➙ All complicated global problems of the geometry are ignored

   [This relies on the assumption that one can decouple gravity.]

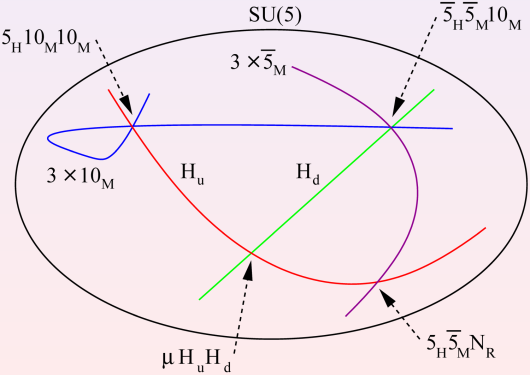
2. Problems in perturbative type-IIB GUT models: It is very difficult to obtain the $\mathrm{SO}(10)$-spinor and the Yukawa coupling $\mathbf{5}_{\mathrm{H}} \cdot \mathbf{10}_{\mathrm{M}} \cdot \mathbf{10}_{\mathrm{M}}$ for the top quark mass in $\mathrm{SU}(5)$ models.
   ➙ Both are easily dealt with in non-perturbative type-IIB / F-theory

3. Geometric "hierarchy" of Vafa's model:

Gravity:	10d	*(bulk)*	B_3
Gauge fields:	8d	*(located on 7-branes)*	S_2
Matter fields:	6d	*(double intersection of 7-branes)*	$S \cap S'$
Interactions:	4d	*(triple intersection of 7-branes)*	$S \cap S' \cap S''$

Vafa et al. II: An $\mathrm{SU}(5)$ toy model

4. The matter content and interactions are determined by the singularity enhancement at the intersections.
   ➙ "Exceptional" enhancements at intersections
5. The doublet-triplet splitting problem can be dealt with by localizing $\mathrm{H_u}$ and $\mathrm{H_d}$ on different branes.



$\mathrm{SU}(5)$ toy model

Summary & Conclusion

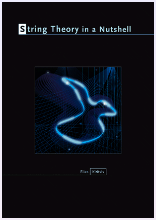
Part I: 4d model building

- "Bottom-up" model building: Using intersecting stacks of D-branes and appropiate O-planes on Calabi-Yau spaces one can construct various (realistic) 4d $\mathcal{N} = 1$ models.
- All ingredients have to be added by hand.
- Global consistency conditions are well understood.

Part II: Non-perturbative aspects

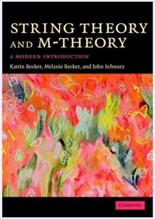
- "Top-down" model building: Almost all properties are encoded in an unified (geometric) structure.
- Many ingredients are required by consistency conditions.
- Global models are difficult to construct explicitly.

Books to start serious reading...



Elias Kiritsis:
"String Theory in a Nutshell", 2007

K. Becker, M. Becker, J. Schwarz:
"String Theory and M-Theory", 2007



Older books - particularly useful for computational details:

- Joseph Polchinski: ***"String Theory"***
  1. ***An Introduction to the Bosonic String***, 1998
  2. ***Superstring Theory and beyond***, 1998
- Green, Schwarz, Witten: ***"Superstring Theory"***
  1. ***Introduction***, 1987
  2. ***Loop Amplitudes, Anomalies and Phenomenology***, 1988
- Lüst, Theisen: ***"Lectures on String Theory"***,1989
  → 2nd revised edition coming 2009