String Theory
A general overview & current “hot” topics

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- 4d model building
- Non-perturbative aspects
- Optional:
  - Vafa’s F-theory GUT model building

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Bosonic string:
- Nambu-Goto and Polyakov action
- Mode expansion and Virasoro algebra
- Interactions and scattering

Superstrings:
- Worldsheet SUSY, spacetime fermions
- Type-I and type-II superstring spectrum
- Spacetime SUSY

Toroidal compactification:
- Circle compactification of the bosonic string
- 2-Torus compactification and modular invariance
- Winding modes and T-duality

D-branes

Chan-Paton factors
Part I

4d model building

- Calabi-Yau compactification
- Orbifolds
- Flux compactification and moduli stabilization
- D-branes and orientifolds
Compactifying the 10d $\mathcal{N} = 2$ type-II superstring on a 6-torus yields (maximal) 4d $\mathcal{N} = 8$ supersymmetry in the effective theory. ➡️ no chiral 4d fermions in toroidal compactifications

New approach: Investigate the effective 10d $\mathcal{N} = 2$ type-II SUGRA and find conditions for 4d minimal supersymmetry.

1 Assume a product space-time $\mathcal{M}_{10} = \mathbb{R}^{1,3} \times K^6$, where $K^6$ is some compact inner space.
  ➡️ 4d Poincaré-invariance: All the fermionic fields must be trivial!
SUSY variations without R-R fluxes

**bosons:** All vanish due to the trivial fermionic fields (parity)

**gravitino:**
\[ \delta_\epsilon \chi_\mu = \nabla_\mu \epsilon - \frac{1}{4} H_{\mu\nu\rho} \gamma^{[\nu\gamma\rho]} \epsilon + (\text{fermions})^2 \]

**dilatino:**
\[ \delta_\epsilon \varphi = (\gamma^\mu \nabla_\mu \Phi) \epsilon + \frac{1}{24} H_{\mu\nu\rho} \gamma^{[\mu\gamma\nu\gamma\rho]} \epsilon + (\text{fermions})^2 \]

One imposes additional simplifications in order to make the SUSY variations of the fermionic fields vanish:

1. Consider a **constant dilaton** field \( \Phi = \Phi_0 \).
2. Consider a **vanishing NS-NS background flux**, i.e. \( H_3 = 0 \).
   - Dilatino equation automatically satisfied.

**Remaining no-flux SUSY variations**

**gravitino:**
\[ \delta_\epsilon \chi_\mu = \nabla_\mu \epsilon \]
On $\mathbb{R}^{1,3} \times K^6$ the equation $\nabla_\mu \epsilon = 0$ splits into two conditions:

1. Constant spinor on $\mathbb{R}^{1,3}$  ⇒ trivial condition.
2. Covariantly constant, nowhere vanishing spinor on compact space $K^6$  ⇒ severe constraints on the metric and topology.

**Theorem of Wang**

$K^6$ is Ricci-flat and Kähler.

**Berger’s holonomy classification**

$K^6$ has $\text{SU}(3)$-holonomy.

What could possibly happen to a tangent vector, when it is parallel transported along a closed loop?

**Holonomy:**

A 6d $\text{SU}(3)$-holonomy manifold is just a Calabi-Yau manifold.

⇒ The internal space $K^6$ must be a Calabi-Yau 3-fold.
In string theory model building only the massless spectrum is usually considered. The primary interest is in the associated effective SUGRA theory obtained by compactification:

- Heterotic string theory on CY 3-fold \( \Rightarrow 4d \mathcal{N} = 1 \) het. SUGRA
- Type-II string theory on CY 3-fold \( \Rightarrow 4d \mathcal{N} = 2 \) type-II SUGRA

\[ \Rightarrow \text{Need to break SUSY further down do } \mathcal{N} = 1. \]
Problem: Non-trivial CY 3-fold metrics are not explicitly known. Only access to topology of the compact space $K^6$.

<table>
<thead>
<tr>
<th>Type-IIA on CY 3-fold</th>
<th>Type-IIB on CY 3-fold</th>
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<tbody>
<tr>
<td>$h^{1,1}$ Abelian vector multiplets</td>
<td>$h^{2,1}$ Abelian vector multiplets</td>
</tr>
<tr>
<td>$h^{2,1} + 1$ hypermultiplets</td>
<td>$h^{1,1} + 1$ hypermultiplets</td>
</tr>
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Hodge numbers: $h^{p,q} := \dim_{\mathbb{R}} H^{p,q}_\bar{\partial}(K^6)$ are the dimensions of the Dolbeault cohomology groups (c.f. Betti numbers).

The massless scalars in those $\mathcal{N} = 2$ multiplets of the effective theory parameterize the geometric moduli space, which is a product space:

$$\mathcal{M}_{\text{geom}}(K^6) = \mathcal{M}^{1,1}(K^6) \times \mathcal{M}^{2,1}(K^6).$$

The expectation values of the massless scalars are determined by the values of the geometric moduli.
Number of massless scalars in effective theory

- type-IIA \( \rightarrow 2h^{1,1} + 4(h^{2,1} + 1) \) scalars
- type-IIB \( \rightarrow 2h^{2,1} + 4(h^{1,1} + 1) \) scalars

Scalars:
- An \( \mathcal{N} = 2 \) hypermultiplet has 4 real scalars,
- an Abelian vector multiplet has 2 real scalars

- Changing the value of the massless scalars does not require any energy, therefore they may take any value.

- Such unconstrained massless scalars give rise to an additional 5th interaction unobserved in nature.
  - ➔ The scalars have to disappear / be fixed.

- But unfortunately in the effective type-II supergravity there is no potential for the massless scalars...
  - ➔ Moduli stabilization problem
Compactifications: A preliminary overview

- Toroidal compactification:
  - Very simple to compute using periodic coordinates.
  - No supersymmetry breaking ($\mathcal{N} = 8$ for type-II)
    ➔ phenomenologically ruled out

- Orbifold compactification:
  - Computations quite similar to toroidal compactifications
  - Singular space-time
  - Breaks supersymmetry to minimal levels ($\mathcal{N} = 2$ for type-II)
    ➔ need further SUSY breaking

- Calabi-Yau compactification:
  - Breaks supersymmetry to minimal levels ($\mathcal{N} = 2$ for type-II)
  - Only access to topological data of the internal space
  - Introduces lots of massless scalars (geometric moduli)
    ➔ need moduli stabilization and further SUSY breaking

➔ need further ingredients: Fluxes & $D_p$-branes & orientifolds
**R-R \( p \)-forms in type-IIA**

- Opp. chirality: \( L|+\rangle \otimes |-_R \Rightarrow C_1, C_3, C_5, C_7, C_9 \)

**R-R \( p \)-forms in type-IIB**

- Same chirality: \( L|+\rangle \otimes |+_R \Rightarrow C_0, C_2, C_4, C_6, C_8, C_{10} \)

- **NS-NS flux:** \( H_3 := dB_2 \) (field-strength of the NS-NS 2-form)

- **R-R fluxes:** \( F := dC - H_3 \wedge C \) (formal sum of all even/odd fluxes)

\[
\begin{align*}
F_0 &= -\ast F_{10} \\
F_2 &= dC_1 \\
F_4 &= dC_3 - H_3 \wedge C_1 \\
F_1 &= dC_0 \\
F_3 &= dC_2 - H_3 C_0 \\
F_5 &= dC_4 - H_3 \wedge C_2
\end{align*}
\]

- Hodge self-duality constraint: \( F_n = (-1)^{\lfloor \frac{n}{2} \rfloor} \ast F_{10-n} \)

- Bianchi identities: \( dH = 0, \quad dF - H \wedge F = 0 \)

- **Basic idea:** In the presence of background fluxes, changing the value of the massless scalars requires energy.
  
  \( \Rightarrow \) Massless scalars obtain dynamically stabilized values
In order to break the supersymmetry to minimal levels as before, one has to consider the SUSY variations of the fields including the R-R fluxes.

**Relevant SUSY variations to first order in $\epsilon$ including R-R fluxes**

- **Bosons:** Vanish due to trivial fermionic fields
- **Gravitino:** 
  \[ \delta_\epsilon \chi_\mu = \nabla_\mu \epsilon + \frac{1}{4} \mathcal{H}_\mu \mathcal{P} \epsilon + \frac{1}{16} e^\Phi \sum_n \mathcal{F}_n \gamma_\mu \mathcal{P}_n \epsilon \]
- **Dilatino:** 
  \[ \delta_\epsilon \varphi = (\partial \phi + \frac{1}{2} \mathcal{H}_\phi) \epsilon + \frac{1}{8} e^\Phi \sum_n (-1)^n (5 - n) \mathcal{F}_n \mathcal{P}_n \epsilon \]

Actually solving the above conditions requires much further techniques:

- Generalized complex geometry ➔ Generalized Calabi-Yau manifolds

This deals with one of the two issues mentioned.

➔ Still need to further break SUSY to $\mathcal{N} = 1$. 
Open strings can have either von-Neumann or Dirichlet boundary conditions. In case of the latter, their space-time position is fixed to a submanifold. → $D_p$-brane.

D-branes are minimally charged under type-II R-R $p$-form fields.

**Electric coupling**

The natural coupling of a $(p+1)$-form to a $p$-brane is given by

$$Q_p \int_{\mathcal{W}_p} A_{p+1}$$

→ electric $p$-brane with charge $Q_p$

**Magnetic coupling**

The magnetic dual to the field strength $F_{p+2} := dA_{p+1}$ is defined by the Hodge dual $\tilde{F} := *F$, i.e.

$$d\tilde{A}_{D-p-3} = \tilde{F}_{D-p-2} = *F_{p+2} = *dA_{p+1}.$$  

→ magnetic $(D - p - 4)$-brane with charge $\tilde{Q}_{D-p-4}$. 
Open string excitations induce a $U(1)$-gauge theory on the $(p+1)$-dimensional $D_p$-brane-worldvolume $\mathcal{W}_p$, which couples to all the lower R-R fields $C_{p+1}, C_{p-1}, C_{p-3}, \ldots$

**Worldvolume actions of D-branes**

- **Dirac-Born-Infeld:**
  
  $$S_{\text{DBI}} = -T_p \int_{\mathcal{W}_p} e^{-\phi} \sqrt{-\det(g + B_2 + 2\pi\alpha' F_2)}$$

- **Chern-Simons:**
  
  $$S_{\text{CS}} = \frac{T_p}{2} \int_{\mathcal{W}_p} C \wedge \text{ch}(F_2) \wedge \underbrace{\sqrt{\hat{A}(R_{T\mathcal{W}_p})}}_{(p+1)-\text{form}} \underbrace{\hat{A}(R_{N\mathcal{W}_p})}_{\text{coupling}}$$

$T_p$ tension of $D_p$-brane
$F_2$ worldvolume $U(1)$ gauge field-strength

**Example:**

$$C \wedge \text{ch}(F_2)|_{8-\text{form}} = C_8 + C_6 \wedge \text{Tr} F_2 + C_4 \wedge \text{Tr}(F_2 \wedge F_2) + \ldots$$
Open strings can stretch between several D-branes.

- A collection of \( N \) coinciding \( D_p \)-branes is called a stack
  - the stack’s worldvolume gauge group enhances to \( U(N) \).
- The branes of a stack can dynamically move apart
  - gauge symmetry breaking \( U(N) \mapsto U(N_1) \times U(N_2) \).
- Intersecting stacks of branes can be used to generate chiral fermions. The states resulting from the intersection transform in the bi-fundamental representation of the stacks’ gauge groups, i.e. \( (\Box_a, \Box_b) \) or \( (\Box_a, \bar{\Box}_b) \).  
  - bi-fundamental (chiral) matter
In a compactification scenario $\mathbb{R}^{1,3} \times K^6$ a $D_p$-brane occupies a $(p-3)$-dimensional submanifold of $K^6$. In the compact directions the total charge and tension must cancel. ➝ Chiral anomalies

According to the Gauß theorem from classical electrodynamics, the force / field-strength in a compact volume is determined by the total charge content.

➤ Tadpole and consistency conditions.

Introduce anti-$D_p$-branes: Same geometric properties as ordinary $D$-branes, but negative charge.

➤ Charge cancellation using $D$-branes and anti-$D$-branes
On the type-II closed superstring, the left- and right-moving are independent of each other except for the level matching ("equal mass") condition. \( \mathcal{N} = 2 \) SUSY

Consider worldsheet orientation-reversing actions.

**Worldsheet parity operator** \( \Omega_P \)

Action of the worldsheet parity operator on the worldsheet: \( \Omega_P : \sigma \mapsto 2\pi - \sigma \)

\( \Rightarrow \) reverses worldsheet orientation

\( \Rightarrow \) Consider the coset space \( \mathcal{M}_{10}/(\Omega_P P) \).

The action of \( P \) on the space-time is usually generated by an involution \( \xi \), i.e. a mapping \( \xi : \mathcal{M}_{10} \rightarrow \mathcal{M}_{10} \) where \( \xi^2 = \text{Id} \).
Orientifolds II: Orientifolding and O-planes

Orientifolding in type-IIA

action: \((-1)^{F_L} \Omega_P \xi\) 

\(O_6\)

- holomorphic volume: \(\xi^* \Omega = \bar{\Omega}\)
- complex structure: \(\xi^* J = -J\)

Orientifolding in type-IIB

action: \((-1)^{F_L} \Omega_P \xi\) or \(\Omega_P \xi\) 

\(O_{5/9}\) or \(O_{3/7}\)

- holomorphic volume: \(\xi^* \Omega = \pm \Omega\)
- complex structure: \(\xi^* J = J\)

- The **fixpoint set** of the orientifold involution \(\xi\) of the space-time is called an orientifold plane.

- Supersymmetry considerations restrict the possible types of O-planes.

\[\text{Tension cancellation using O-planes.}\]
“Bottom up” model building

- **Example:** Orientifolding the \((oriented, closed)\) type-IIB superstring yields the \((unoriented, open/closed)\) type-I superstring.

Calabi-Yau orientifold model building 1-0-1

1. Calabi-Yau geometry ensures 4d \(\mathcal{N} = 2\) supersymmetry.
2. Adding \(D_p\)-branes and O-planes breaks SUSY to 4d \(\mathcal{N} = 1\) and provides the means of \(U(n), SO(n)\) or \(Sp(n)\) gauge theory.
3. Background fluxes can be used to stabilize the moduli.

→ IIA and IIB model building with 4d \(\mathcal{N} = 1\) eff. theories

In principle, we now have all the neccessary tools for perturbative “bottom-up” 4d model building.

→ What about the non-perturbative side?
Part II

Non-perturbative aspects

- Duality web: T-, S- and U-duality
- M-theory
- F-theory as non-perturbative type-IIB theory
### Five consistent 10d perturbative superstring theories

<table>
<thead>
<tr>
<th>Theory</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>open and closed strings in bulk, SO(32) gauge symmetry</td>
</tr>
<tr>
<td>IIA</td>
<td>closed strings in bulk, open strings ending on D-branes, non-chiral</td>
</tr>
<tr>
<td>IIB</td>
<td>closed strings in bulk, open strings ending on D-branes, chiral</td>
</tr>
<tr>
<td>HE</td>
<td>only closed strings in bulk, $E_8 \times E_8$ gauge symmetry</td>
</tr>
<tr>
<td>HO</td>
<td>only closed strings in bulk, SO(32) gauge symmetry</td>
</tr>
</tbody>
</table>

The five superstring theories are related via numerous dualities:

1. **T-duality**: The radius $R$ of a compact direction with the topology of a circle is mapped to $\frac{1}{R}$ and “wrapped” string states will be exchanged with high-momentum string states.

$$ R \leftrightarrow \frac{1}{R} \quad \text{exchange of state types} \quad \widehat{n} \leftrightarrow \widehat{w} $$
Examples of T-duality:

- Type-IIA on radius $R$ is T-dual to type-IIB on radius $\frac{1}{R}$
- Heterotic $E_8 \times E_8$-string is T-dual to the heterotic $SO(32)$-string.

$\Rightarrow$ T-duality relates the small to the large dimensions.

S-duality: Inverts the string coupling, i.e. $g_s \leftrightarrow \frac{1}{g_s}$.

[$\text{The string coupling depends on the dilaton: } g_s := e^{-\Phi} \iff \Phi \mapsto -\Phi.$]

Examples of S-duality:

- Type-I superstring is S-dual to the heterotic $SO(32)$-string
- Type-IIA superstring is S-dual to the heterotic $E_8 \times E_8$-string
- The type-IIB superstring is self-dual under S-duality!

$\Rightarrow$ S-duality relates the weak and strong coupling regime.

U-duality: combination of S- and T-duality in the context of M-theory.
Witten (1995): “If all the perturbative superstring theories are dual to each other, there should be a common non-perturbative origin!”

⇒ M-theory ⇒ sparked the “2nd superstring revolution”

Basic idea: The 5 superstring theories should be certain limiting cases of M-theory.

Main problem: As yet there is no microscopic formulation of M-theory, i.e. there’s no such thing like a “M-theory lagrangian”.

Different approaches:

1. Formulation as the non-perturbative origin of type-IIA superstring theory
2. “Matrix model” formulation as a membrane theory
3. UV-completion of the 11d $\mathcal{N} = 1$ SUGRA
M-Theory II: Type-IIA at strong coupling

- Important observation: The effective 10d type-IIA SUGRA theory is equivalent to the dimensional reduction of 11d SUGRA.

Lowest order 11d $\mathcal{N} = 1$ and type-IIA supergravity (bosonic part)

dimensional reduction: ignore field’s dependency on 11th coord.; $G_4 := d\hat{C}_3$

\[
S_{11d}^{\text{bos}} = \frac{1}{2\kappa^2} \int d^{11}x \sqrt{\tilde{g}} \left[ \tilde{R} - \frac{1}{48} |G_4|^2 \right] + \frac{1}{4\kappa^2} \int \hat{C}_3 \wedge G_4 \wedge G_4
\]

\[
S_{\text{IIA}}^{\text{bos}} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{g} \left[ e^{-2\Phi} \left( R + 4(\nabla \Phi)^2 - \frac{|H_3|^2}{12} \right) - \frac{|F_4|^2}{48} - \frac{|F_2|^2}{2} \right]
\]

(String frame) + $\frac{1}{4\kappa^2} \int B_2 \wedge dC_3 \wedge dC_3$

R-R kinetic terms do not couple to the dilaton

- In the dimensional reduction, the diagonal component $\tilde{g}_{11,11}$ of the 11d metric becomes the string coupling $g_s = e^{-\Phi}$

$\quad$ Type-IIA apparently “grows” a 11th dimension for $g_s \to \infty$
The 11d 3-form field $\hat{C}_3$ naturally couples to 2-branes → M2-branes

M2-branes are supposedly the “fundamental” objects of M-theory, just like strings are the fundamental objects of (perturbative) string theory.

In matrix theory one aims to find a direct quantization of such “membranes” in order to find a direct description of M-theory.

- Quantization conditions seem to be too restrictive
- Classification of 3-manifolds is vastly more difficult compared to the classification of 2-manifolds (string worldsheet topologies)

For example, the Poincaré conjecture (which states that every simply connected, compact 3-manifold (without boundary) is homeomorphic to the 3-sphere) was first solved in 2003 by Grigori Perelman (and led to his - rejected! - Fields Medal in 2006)

→ Many unsolved problems in finding a proper formulation of M-theory...
F-Theory I: Enhanced S-duality of type-IIB

Lowest order of 10d $\mathcal{N} = 2$ type-IIB supergravity (bosonic part)

$$S_{\text{IIB}}^{\text{bos}} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{g} \left[ R - \frac{1}{2} \frac{\partial S \partial \bar{S}}{(\text{Im } S)^2} - \frac{|G_3|^2}{12} - \frac{|F_5|^2}{240} \right]$$

(Einstein frame)

$$+ \frac{1}{2i} \int C_4 \wedge G_3 \wedge \bar{G}_3 \quad \text{where} \quad G_3 := i \frac{F_3 + SH_3}{\sqrt{\text{Im } S}}$$

- In case of type-IIB the $S$-duality group is enhanced to $SL(2; \mathbb{R})$
  - Define the (complex) axio-dilaton $S := C_0 + ie^{-\phi}$ scalar field.
  - Some matrix $\Lambda := \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2; \mathbb{R})$ acts on the fields

  $$S \mapsto \Lambda S := \frac{aS + b}{cS + d} \quad \left( \begin{array}{c} B_2 \\ C_2 \end{array} \right) \mapsto (\Lambda^{-1})^t \left( \begin{array}{c} B_2 \\ C_2 \end{array} \right)$$

- Due to the Dirac quantization condition, the group is reduced to integer coefficients $\Rightarrow SL(2; \mathbb{Z})$ in quantum theory
Vafa (1996): Since $\text{SL}(2; \mathbb{Z})$ is the torus amplitude’s modular group, one should interpret the axio-dilaton value at each space-time point to parameterize a 2-torus complex structure and geometry.

- 12d total space $X \xrightarrow{\pi} M$ elliptically-fibered over 10d space-time $M$
- Due to SUSY constraints one takes an elliptically-fibered compact space $X_4$ with a Calabi-Yau 3-fold base $B_3$.
  - Usually: Elliptically-fibered Calabi-Yau 4-fold $X_4 \xrightarrow{\pi} B_3$
  - Sometimes: Elliptically-fibered $\text{Spin}(7)$-holonomy manifold $Y \xrightarrow{\pi} B_3$
- Consider space-time varying dilaton backgrounds with string coupling $g_s$ being weak and strong
  ➤ F-theory = non-perturbative type-IIB
The R-R scalar $C_0$ couples magnetically to D7-branes.

Consider a closed loop $\gamma$ in the 2d transverse space to the 8d D7-worldvolume. Walking along $\gamma$, the value of $S$ changes value!

- **Monodromy:** $S \mapsto S + 1$
- **Axion-dilaton singularity** at the position of the D7-brane!
- **Singularity** in the elliptic fibre!

### F-theory model building 1-0-1

1. Take an elliptically-fibered compact space $X_4 \xrightarrow{\pi} B_3$.
   - **Stage:** $\mathcal{X} = \mathbb{R}^{3,1} \times X_4 \xrightarrow{\text{pr}_1 \times \pi} \mathcal{M} = \mathbb{R}^{3,1} \times B_3$
2. 7-branes are located at those points in space-time, where the elliptic fibre degenerates.
3. Non-perturbative type-IIB with space-time varying $g_s$ is described.
Recent Developments

- More on F-theory
- F-theory GUTs with exceptional gauge groups (Vafa et al. '08)
The degeneration of the elliptic fibre in mathematically understood in terms of Enriques-Kodaira singularities.

Singularities of Kodaira type $I_1$ ("double point")

Let $\alpha, \beta \in H_1(T^2) \cong \pi_1(T^2) \cong \mathbb{Z} \oplus \mathbb{Z}$ denote the two generators corresponding to the two independent circles on a torus.

$\rightarrow$ torus degeneration: $1$-cycle $p\alpha + q\beta$ collapses

The objects located at such degeneration points are $(p, q)$ 7-branes.

$(p, q)$ 7-branes can be treated as the image of ordinary D7-branes under appropriate $\text{SL}(2; \mathbb{Z})$-transformations.

- Ordinary D7-branes are identified as $(1, 0)$ 7-branes
- O7-planes can be resolved into certain pairs of $(p, q)$ 7-branes

In the same spirit one defines $(p, q)$-strings connecting the 7-branes.
Singularities of other Kodaire type: ➔ ADE singularities

- Consider an elliptically-fibered CY 4-fold \( X_4 \xrightarrow{\pi} B_3 \) and let \( S_2 \subset B_3 \) denote the singularity locus where the fibre degenerates.
- Let \( K_S \xrightarrow{\pi K} S \) denote the canonical line bundle and \( N_S \xrightarrow{\pi N} S \) the normal line bundle of \( S \subset B \).
- The total space of \( X|_S \xrightarrow{\pi} S \) looks like an isolated ADE surface singularity fibered over \( S \).

### ADE singularities

- \( A_n \) \( f = y^2 - x^2 - z^{n+1} \) \( \text{SU}(n+1) \)
- \( D_n \) \( f = y^2 - x^2z - z^{n-1} \) \( \text{SO}(2n) \)
- \( E_6 \) \( f = y^2 - x^3 - z^4 \) \( E_6 \)
- \( E_7 \) \( f = y^2 - x^3 - xz^3 \) \( E_7 \)
- \( E_8 \) \( f = y^2 - x^3 - z^5 \) \( E_8 \)

Let \( x, y, z \) be the fibre coordinates (sections) of \( K_S^a \oplus K_S^b \oplus N_S \rightarrow S \).

The subset \( \{ f(x, y, z) = 0 \} \) of this bundle’s total space describes \( X|_S \xrightarrow{\pi} S \).
The ADE singularity determines the worldvolume gauge group of the corresponding ADE 7-brane.

Correspondence to the perturbative constructions:

- \( SU(n+1) \): Realized by stacks of \( n+1 \) coincident D7-branes.
- \( SO(2n) \): Realized by D7-branes on top of O7-planes.
- \( E_6, E_7, E_8 \): Can be realized by using specific arrangements of \((p,q)\) 7-branes which are connected by appropriate \((p,q)\)-string networks.

The F-theory approach naturally yields exceptional gauge groups!

Given two ADE 7-branes on \( S, S' \subset B \) with gauge groups \( G, G' \) the gauge group / singularity enhances over the intersection: \( G_\Sigma \supset G \times G' \).
The basic ideas of the work of Vafa et al:

1. Consider only **local models**, i.e. the elliptically-fibered CY 4-fold is only considered in a small neighbourhood of the singular locus $S \subset B$.
   ➤ All complicated global problems of the geometry are ignored.
   [This relies on the assumption that one can decouple gravity.]

2. **Problems in perturbative type-IIB GUT models**: It is very difficult to obtain the $SO(10)$-spinor and the Yukawa coupling $5_H \cdot 10_M \cdot 10_M$ for the top quark mass in $SU(5)$ models.
   ➤ Both are easily dealt with in non-perturbative type-IIB / F-theory.

3. **Geometric “hierarchy”** of Vafa’s model:

   - **Gravity**: 10d *(bulk)*
   - **Gauge fields**: 8d *(located on 7-branes)*
   - **Matter fields**: 6d *(double intersection of 7-branes)*
   - **Interactions**: 4d *(triple intersection of 7-branes)*
The matter content and interactions are determined by the singularity enhancement at the intersections.

→ “Exceptional” enhancements at intersections

The doublet-triplet splitting problem can be dealt with by localizing $H_u$ and $H_d$ on different branes.
Part I: 4d model building

- “Bottom-up” model building: Using intersecting stacks of D-branes and appropriate O-planes on Calabi-Yau spaces one can construct various (realistic) $4d \, \mathcal{N} = 1$ models.
- All ingredients have to be added by hand.
- Global consistency conditions are well understood.

Part II: Non-perturbative aspects

- “Top-down” model building: Almost all properties are encoded in an unified (geometric) structure.
- Many ingredients are required by consistency conditions.
- Global models are difficult to construct explicitly.
Books to start serious reading...

Elias Kiritsis: 
“String Theory in a Nutshell”, 2007

K. Becker, M. Becker, J. Schwarz: 

Older books - particularly useful for computational details:

- Joseph Polchinski: “String Theory”
  1. An Introduction to the Bosonic String, 1998

- Green, Schwarz, Witten: “Superstring Theory”
  1. Introduction, 1987
  2. Loop Amplitudes, Anomalies and Phenomenology, 1988

  → 2nd revised edition coming 2009