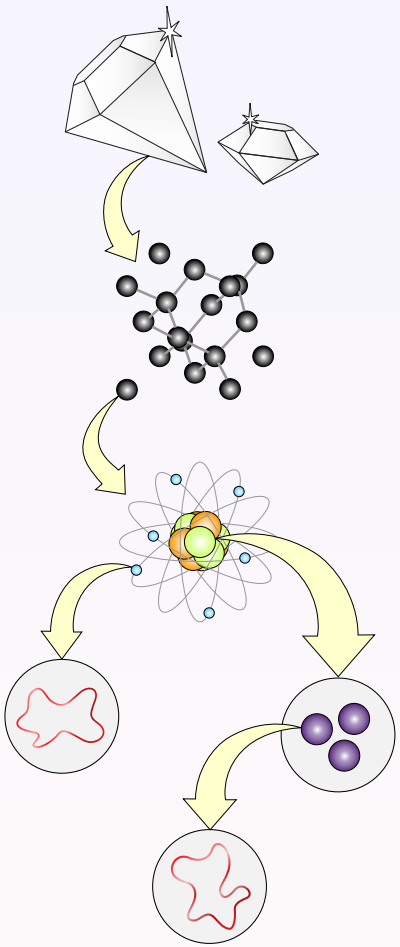
Aspects in F-Theory Model Building



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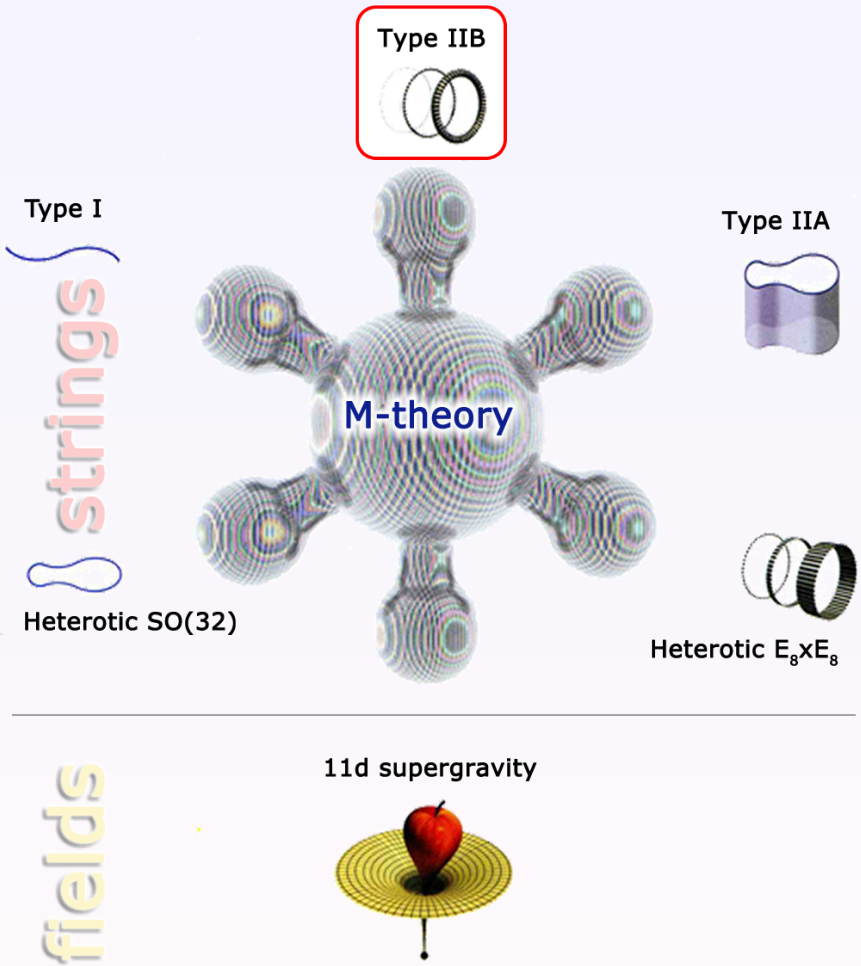
IMPRS Young Scientists Workshop (Ringberg)

— July 28th, 2010 —

M-theory



GOAL: **Construct a stringy GUT!**

5 perturbative 10d string theories

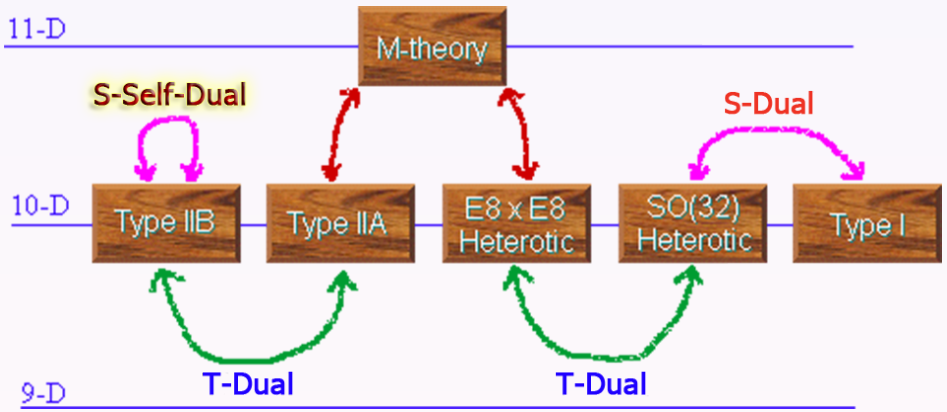
- **Heterotic:** Closed strings, either $E_8 \times E_8$ or $SO(32)$.
- **Type I:** Open & closed strings, gauge group $SO(32)$.
- **Type II:** Closed strings, no non-abelian gauge group.
  - **IIA:** non-chiral
  - **IIB:** chiral

Different limits of **M-theory**, a non-perturbative completion of Type IIA string theory.

The string duality web

String dualities

- **T-duality:** relates small compact dimensions to large ones
  (radius $R \leftrightarrow \frac{1}{R}$, winding number / momentum: $w \leftrightarrow n$)
- **S-duality:** relates weak-coupling to strong-coupling regimes
  (string coupling $g_s \leftrightarrow \frac{1}{g_s}$)



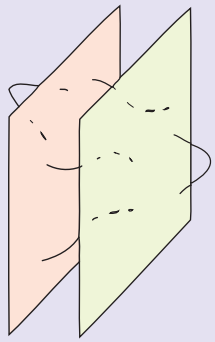
Problem:
M-theory poorly understood.

A more accessible framework exists for non-perturbative Type IIB theory.

➙ **F-theory**

D-branes and non-abelian gauge groups

D-branes in Type II string theory...



- ...are **higher-dimensional objects**
- ...have **open strings** ending on them
- ...carry a $U(1)$ **worldvolume gauge theory**
- ...can **intersect**, giving additional states

terminology: 7-brane = 7 spatial dimensions, i.e. 8d worldvolume

A stack of n D-branes carries an $U(n)$ worldvolume gauge theory.
In orientifold settings also $SO(n)$ and $Sp(n)$.

➙ Try a D-brane GUT theory...

Unfortunately: Important Yukawa couplings and states/representations are missing, doublet-triplet splitting remains an issue. **Nothing gained...**

Aspect: Instantons in string theory

There are non-perturbative ingredients in string theory:

loop levels ➞ **perturbative** contributions: $\propto g_s^l$

instantons ➞ **non-perturbative** contributions: $\propto \exp\left(-\frac{1}{g_s^2}\right)$

Important when all larger perturbative contributions are absent.

Instantons in string theory

Objects localized in space-time, i.e. wrapping the internal space:

- **Worldsheet instantons**: Strings wrapping 2-cycles
- **D-brane instantons**: D_p-branes wrapping p-cycles

Instantons in gauge theory

Self-dual configurations, which are local minima of the system:

4d: $\underbrace{F = \pm \star F}$

(anti-)self-dual field strength

D-brane instantons

Some properties:

- Generation of certain **superpotential contributions**
  (independant from the gauge degrees of freedom)
  ➙ Crucial property for some attempts of **moduli stabilization**!
- **Breaking of global symmetries**: ➙ **matter couplings**
- Exponential suppression determined by the size of the wrapped cycle.
  ➙ Geometry-controlled contribution

What is an E3-brane instanton in IIB?

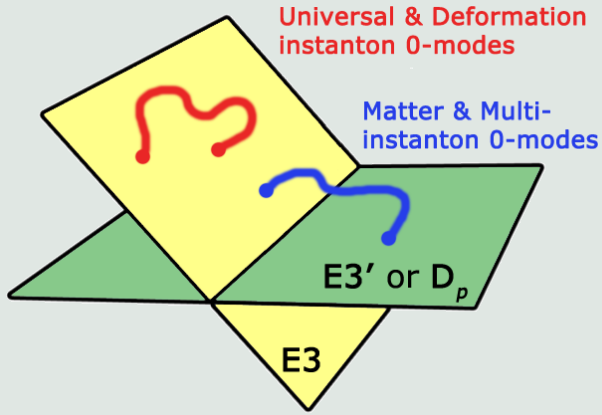
4-dimensional "volume" wrapped around a Euclidean 4-cycle of the internal geometry. Appears as a point in the 4d flat space-time.

D-brane instanton 0-modes

Ultimately, one is interested in the effective 4d theory resulting from the massless Kaluza-Klein modes ("0-modes") of the compactification.

Instanton 0-modes

- **Universal 0-modes**: E3—E3 strings
- **Deformation 0-modes**: From deformations of the E3-brane geometry.
- **Charged / Matter 0-modes**: E3—D_p strings from intersections with D_p branes.
- **Multi-instanton 0-modes**: E3—E3' strings from intersections of multiple E3-instantons.



Big issue: Is there a non-perturbative perspective?

➙ **F-theory**

F-theory basics I

IIB string theory comes with 2 scalar fields: the axion C_0 & the dilaton ϕ, which can be used to **parameterize the geometry of a torus**.

➙ "Shape of torus = value of 2 background scalars"

As we move around in 10d space-time, the value of the fields / shape of the torus varies.

➙ Put together, this gives an **elliptically-fibered 12d space**.

roughly: *locally* the space looks like (10d base)×(torus fibre)

In short: "Geometrization" of the 10d space-time and 2 background fields.

What is F-theory?

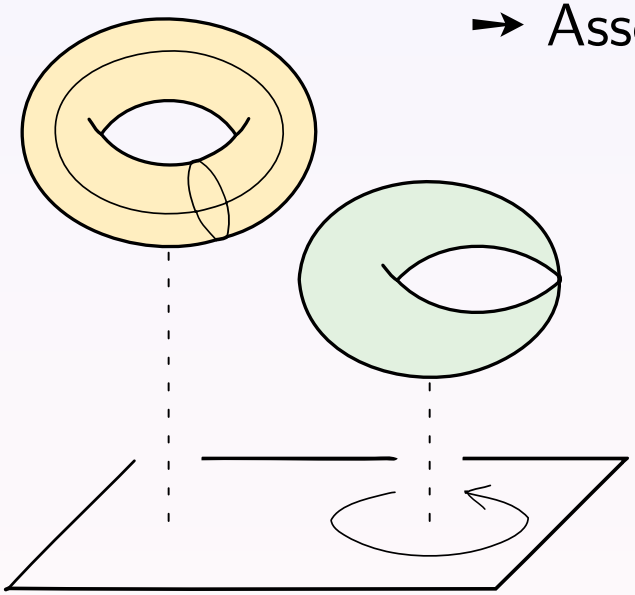
F-theory is the "uplifted theory" defined on this ell.-fib. 12d space, which is understood as a **non-perturbative completion of type-IIB string theory**.

F-theory basics II

Consider a 7-brane (recall: 8d worldvolume!) in 10d space-time, leaves 2 transverse dimensions. Now consider walking around the 7-brane in this "transverse plane": **The value of the axion field changes!**

➙ **Singularity** in the axion field where the D-brane sits!!!

➙ Associated **torus singular as well!**



The elliptic fibration encodes...

- ...2 scalar fields: axion & dilaton
- ...the coupling constant
- ...the location of 7-branes

F-theory basics III

Question: Given an elliptically-fibered 12d space, what can happen?

In fact, besides the location of the 7-branes, **the elliptic fibration also encodes the world-volume gauge group**.
Technical description in terms of a Weierstrass model:

$$y^2 = x^3 + xz^4 \cdot f(u_i) + z^6 \cdot g(u_i)$$

x, y, z fibre coordinates
f, g polynomials

➙ "Kodaira classification of singular fibres" depends on f and g

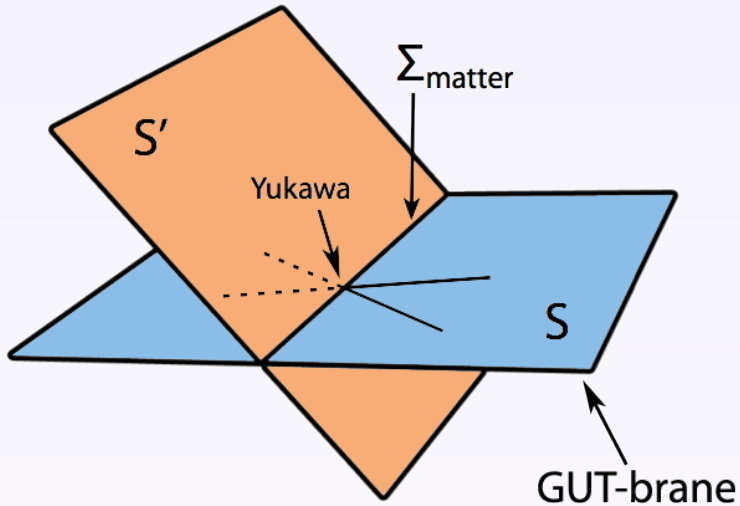
Worldvolume gauge groups

plain D-branes:	$U(n)$
orientifolds:	$+ SO(n)$, $Sp(n)$
F-theory:	$+ \boldsymbol{E_6}$, $\boldsymbol{E_7}$, $\boldsymbol{E_8}$, $\boldsymbol{F_4}$, $\boldsymbol{G_2}$

F-theory GUTs

Intersecting D-branes provide bifundamental matter states. Those come from the **decomposition of the adjoint representation of the "intersection group"**.

F-theory: Exceptional groups!



Now select an $\mathrm{SU}(5)$-**GUT-7-brane** as the "stage".

Consider multiple D-branes intersecting:

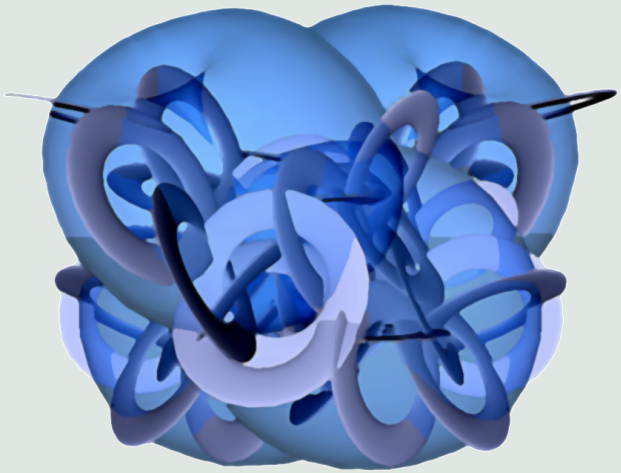
- **Matter curves:** 1 brane intersecting the GUT brane ➞ curve
- **Interactions:** 2 branes intersecting the GUT brane ➞ point
- **Yukawa couplings:** 3 branes intersecting the GUT brane ➞ point

Hitting a brick wall?

Unfortunately, those **local** properties have to be embedded in a full **global** model to apply consistency considerations.

Short list of conditions

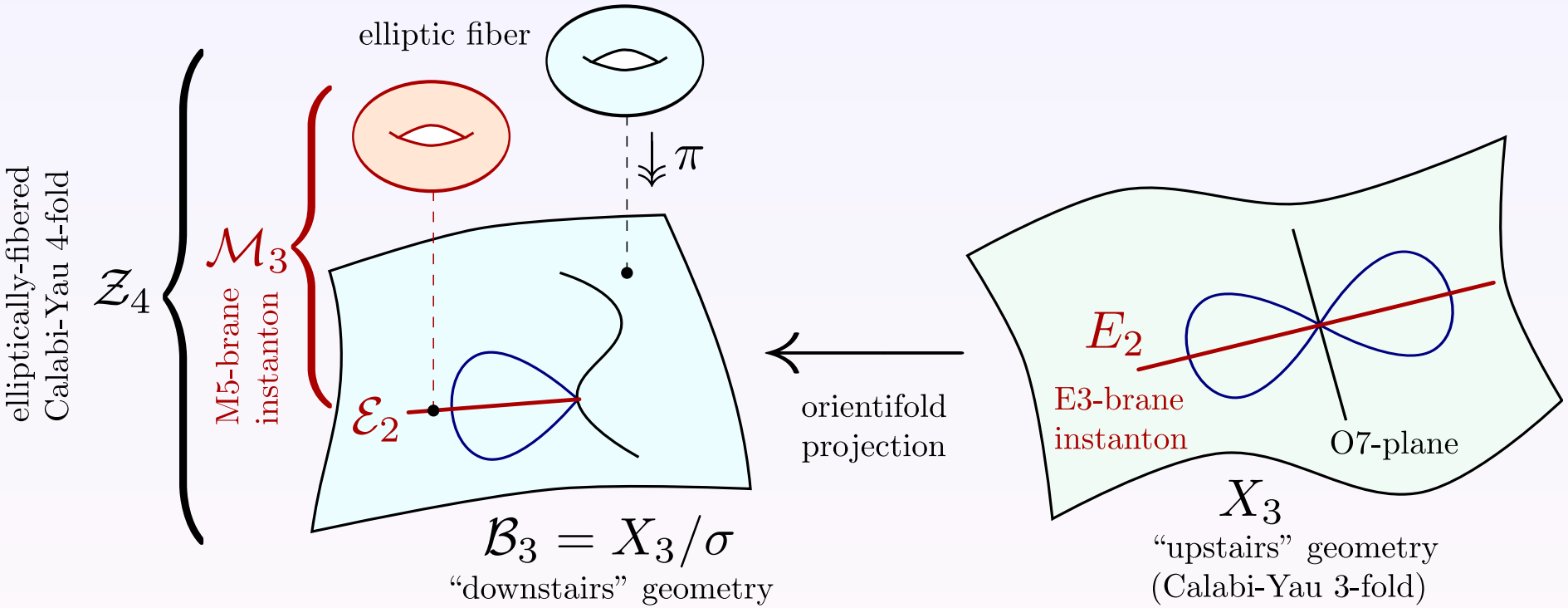
- A **compact** Calabi-Yau 4-fold (8d space), **elliptically-fibered** over a Kähler 3-fold base, such that the elliptic fibration...
  - ...provides a suitable **GUT 7-brane**
  - ...provides further **7-branes intersections** with the GUT 7-brane just right in order to satisfy the phenomenological constraints



➙ Small steps: Try to uplift a known Type-IIB model.

Uplifting

Consider a familiar Type IIB orientifold setting with D3-brane instantons:



Biggest issue: 0-mode counting for M5-brane instantons in F-theory?

Matching the 0-modes & Conclusion

0-mode matching

$h^{0,3}(\mathrm{M5})$

$h^{0,2}_{-}(\mathrm{E3})$

$h^{0,2}_{+}(\mathrm{E3}) \longrightarrow h^{0,2}(\mathrm{M5})$

$h^{0,1}_{-}(\mathrm{E3})$

$h^{0,1}_{+}(\mathrm{E3}) \longrightarrow h^{0,1}(\mathrm{M5})$

$h^{0,0}_{-}(\mathrm{E3})$

$h^{0,0}_{+}(\mathrm{E3}) \longrightarrow h^{0,0}(\mathrm{M5})$

Matching the 0-modes requires sophisticated computational techniques.

➙ Spawned a **math project** of its own:

One result of all those efforts:
Some of the IIB instanton 0-modes are actually non-perturbatively lifted ("recombined") when one moves away from the perturbative IIB Sen limit of F-theory!

➙ **Refined perspective on instantons**, which—to emphazise again—are crucial for generating couplings and moduli stabilization.

coming next: **Bavarian Buffet...**