Computational Tools for String Phenomenology

OR: The importance of cohomology group dimensions

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Contents

- String pheno & cohomology
- Toric geometry 1-0-1
- Algorithm & Applications
- **cohomCalg** implementation

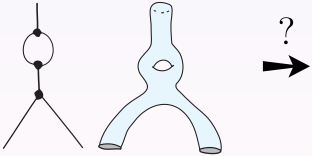
Boston (Northeastern University) — June 13, 2011

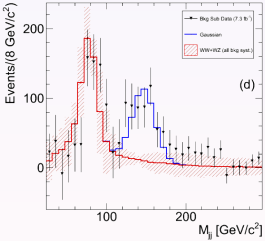
with R. Blumenhagen, T. Rahn, H. Roschy

- **Algorithm:** arXiv:1003.5217
- **Proofs:** arXiv:1006.2392 & S.-T. Jow: arXiv:1006.0780
- **Applications:** arXiv:1010.3717

Part 1

- **Motivation**:
  String phenomenology & cohomology groups





In Type II string models **intersecting D-branes** and **gauge fluxes** are required in order to generate chiral matter.

Example: $U(1)$ gauge flux on D7s

D_a, D_b two stacks of D7s intersecting over curve $C = D_a \cap D_b$, matter in the bifundamental $(\bar{N}_a, N_b)$ is counted by

$$H^i(C; L_a^\vee \otimes L_b \otimes K_C^{\frac{1}{2}}).$$

The chiral index gives the net number of chiral states:

$$\begin{aligned} I_{ab}^{\text{loc}} &= \chi(C; L_a^\vee \otimes L_b \otimes K_C^{\frac{1}{2}}) \\ &\overset{\text{RRH}}{=} -\int_{\mathcal{X}} [D_a] \wedge [D_b] \wedge (c_1(L_a) - c_1(L_b)) \end{aligned}$$

[Blumenhagen-Körs-Lüst-Stieberger '06, Blumenhagen-Braun-Grimm-Weigand '08]

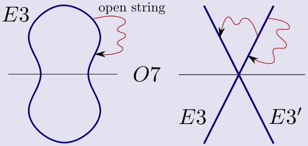
D-brane instanton counting

Euclidean D-brane instantons (E-branes) are **entirely wrapped around the compactified dimensions**, i.e. pointlike from the 4d perspective.

Example: Zero modes counting for E3-brane instanton

zero modes	number
(X_μ, θ_α)	1
$\bar{\tau}_{\dot{\alpha}}$	0
γ_α	$h_+^{1,0}(E)$
$(w, \bar{\gamma}_{\dot{\alpha}})$	$h_-^{1,0}(E)$
χ_α	$h_+^{2,0}(E)$
$(c, \bar{\chi}_{\dot{\alpha}})$	$h_-^{2,0}(E)$

Self-invariant
$O(1)$ instanton



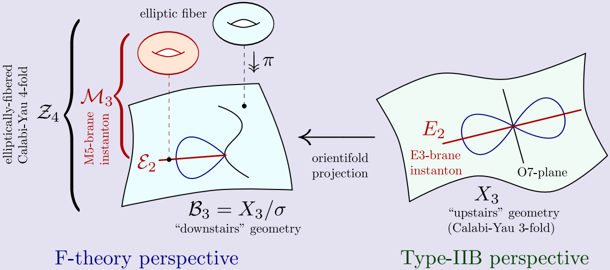
zero modes	number
X_μ	1
θ_α	1
$\bar{\tau}_{\dot{\alpha}}$	1
$(w, \gamma_\alpha, \bar{\gamma}_{\dot{\alpha}})$	$h^{1,0}(E)$
$(c, \chi_\alpha, \bar{\chi}_{\dot{\alpha}})$	$h^{2,0}(E)$

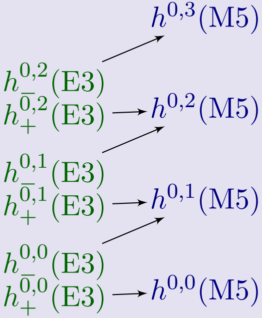
Generic
$U(1)$ instanton

[Blumenhagen-Cvetic-Kachru-Weigand '08]

Type IIB E3-brane instanton zero modes can be matched to vertical **M5-brane instantons in F-theory**.

Example: Zero modes matching for E3/M5-brane instantons





[Blumenhagen-Jurke-Collinucci '10]

In heterotic string modes on the Calabi-Yau 3-fold $\mathcal{X}$ a (stable, holomorphic) **vector bundle breaks the gauge group** $E_8 \times E_8$ or $SO(32)$.

Example: $SU(5)$ model in heterotic string theory

Let $\mathcal{X} = \mathbb{P}^4[5]$ be the quintic Calabi-Yau 3-fold. The monad

$$0 \longrightarrow V \hookrightarrow \mathcal{O}_{\mathcal{X}}(2)^{\oplus 5} \oplus \mathcal{O}_{\mathcal{X}}(1)^{\oplus 5} \twoheadrightarrow \mathcal{O}_{\mathcal{X}}(3)^{\oplus 5} \longrightarrow 0$$

describes an $SU(5)$-bundle, yielding a $SU(5)$ GUT of the $E_8 \times E_8$ heterotic string (after modding out $\mathbb{Z}_5 \times \mathbb{Z}_5$). The **particle spectrum** is given by:

$$\underbrace{\begin{aligned} \eta_{10} &= h^1(\mathcal{X}; V) \\ \eta_{\overline{10}} &= h^1(\mathcal{X}; V^*) \end{aligned}}_{Q \oplus U \oplus E} \qquad \underbrace{\begin{aligned} \eta_{5} &= h^1(\mathcal{X}; \Lambda^2 V) \\ \eta_{\overline{5}} &= h^1(\mathcal{X}; \Lambda^2 V^*) \end{aligned}}_{D \oplus L}$$

[Anderson-Gray-He-Lukas '09]

In all mentioned examples the phenomenological aspects are ultimately determined by the dimension of vector-bundle-valued cohomology groups.

In fact: Most vector bundles are constructed as monads from line bundles

$$0 \longrightarrow V \hookrightarrow \bigoplus_j \mathcal{O}_{\mathcal{X}}(b_j) \twoheadrightarrow \bigoplus_k \mathcal{O}_{\mathcal{X}}(c_k) \longrightarrow 0$$

Examples of monads

- The **tangent bundle** $T_{\mathcal{X}}$ of toric varieties is a monad.
- The **vector bundle moduli** can be computed from $\mathrm{End}(V) \cong V \otimes V^*$.

Important tool: **Exactness of sequences**, i.e. $\mathrm{image}(f_i) = \mathrm{kernel}(f_{i+1})$.

If a sequence is exact, the location of dimension-0 spaces often suffices to determine isomorphisms, which makes computations a lot easier.

From short exact sequences of sums of line bundles one considers the **induced long exact sequences** of the cohomology, e.g.:

$$0 \longrightarrow \mathcal{O}_{\mathcal{X}}^{\oplus r} \hookrightarrow \bigoplus_k \mathcal{O}_{\mathcal{X}}(D_k) \twoheadrightarrow T_{\mathcal{X}} \longrightarrow 0$$

$$\Downarrow$$

$$\begin{aligned}
0 \longrightarrow H^0(\mathcal{X};\mathcal{O}_{\mathcal{X}})^{\oplus r} &\longrightarrow \bigoplus_k H^0(\mathcal{X};\mathcal{O}_{\mathcal{X}}(D_k)) \longrightarrow H^0(\mathcal{X};T_{\mathcal{X}}) \longrightarrow \\
\longrightarrow H^1(\mathcal{X};\mathcal{O}_{\mathcal{X}})^{\oplus r} &\longrightarrow \bigoplus_k H^1(\mathcal{X};\mathcal{O}_{\mathcal{X}}(D_k)) \longrightarrow H^1(\mathcal{X};T_{\mathcal{X}}) \longrightarrow \\
\longrightarrow H^2(\mathcal{X};\mathcal{O}_{\mathcal{X}})^{\oplus r} &\longrightarrow \bigoplus_k H^2(\mathcal{X};\mathcal{O}_{\mathcal{X}}(D_k)) \longrightarrow H^2(\mathcal{X};T_{\mathcal{X}}) \longrightarrow \ldots
\end{aligned}$$

From a phenomenologists point of view, everything boils down to the computation of line bundle-valued cohomology group dimension $h^i(\mathcal{X}; L_{\mathcal{X}})$. What to do?

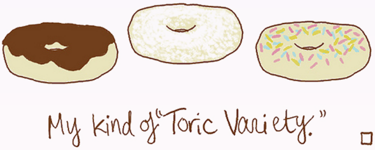
Known methods

- **Isomorphisms**: If you can find isomorphisms to spaces with known cohomology, you don't have to compute anything.
- **Spectral sequences**: The method of spectral sequences allows to compute the cohomology of general spaces, but it is extremely laborious to work with.

$\rightarrow$ Find a new method to compute line bundle cohomology!

Part 2

- Basics of **toric geometry**



□

Reminder: Complex projective space

$\mathbb{P}^n$ is constructed as a quotient space. The coordinates $x_0, \ldots, x_n$ of $\mathbb{C}^{n+1}$ are subject to the $\mathbb{C}^\times$-action

$$(x_0, \ldots, x_n) \mapsto (\lambda x_0, \ldots, \lambda x_n) \quad \text{for all } \lambda \in \mathbb{C}^\times,$$

if the origin $0 \in \mathbb{C}^{n+1}$ is removed, thus $\mathbb{P}^n = \dfrac{\mathbb{C}^{n+1} - \{0\}}{\mathbb{C}^\times}$.

Weighted projective space

Generalize the $\mathbb{C}^\times$-action by changing the powers of λ:

$$(x_0, \ldots, x_n) \mapsto (\lambda^{Q_0} x_0, \ldots, \lambda^{Q_n} x_n) \quad \text{for all } \lambda \in \mathbb{C}^\times,$$

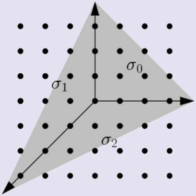
giving $\mathbb{P}^n_{Q_0,\ldots,Q_n}$. Note that $\mathbb{P}^n = \mathbb{P}^n_{1,\ldots,1}$.

Toric variety

Use several $\mathbb{C}^\times$-actions with different weights. The powers are given by charges Q_k^a such that

$$(x_0, \ldots, x_n) \mapsto \Big((\lambda_1^{Q_0^1} \cdots \lambda_r^{Q_0^r}) x_0, \ldots, (\lambda_1^{Q_n^1} \cdots \lambda_r^{Q_n^r}) x_n \Big) \quad \text{for } \lambda_i \in \mathbb{C}^\times,$$

defines a $(C^\times)^r$-action. Instead of just the origin, a set Z has to be removed from $\mathbb{C}^{n+r}$. The **toric variety** is then



$$\mathcal{X} = \frac{\mathbb{C}^{n+r} - Z}{(\mathbb{C}^\times)^r}$$

There is an alternative, entirely combinatorial perspective on toric varieties that makes this kind of geometry ideally suited for algorithmic treatments.

Stanley-Reisner ideal

The Stanley-Reisner ideal encodes sets of coordinates x_i that are **NOT** allowed to vanish simultaneously. It can be generated by squarefree monomials $\mathcal{S}_j$, which are the products of those coordinates:

$$\begin{aligned}\mathrm{SR}(\mathcal{X}) &= \left\{x_{k_1}\cdots x_{k_p} : \{x_{k_1},\ldots,x_{k_p}\}\in Z\right\} \\ &= \langle \mathcal{S}_1,\ldots,\mathcal{S}_t\rangle\end{aligned}$$

Example: $\mathrm{SR}(\mathbb{P}^2) = \langle x_0x_1x_2\rangle$ encondes the removed origin.

Divisors on toric varieties

A divisor on a toric variety is basically a formal sum of codimension-1 hypersurfaces. With respect to the chosen coordinates $x_0,\ldots,x_n$ of $\mathcal{X}$, one frequently uses the divisors $D_i := \{x_i = 0\}\subset\mathcal{X}$.

Line bundles

From the projective powers Q_k^a of a toric variety (called GLSM charges in a different context) one can directly read off the classes of the divisors D_i.

Example: $\mathbb{P}^2 = \mathbb{P}^2_{1,1,1}$ has just one projective relation and each power is $Q_k^1 = 1$. Thus $[D_1] = [D_2] = [D_3] = H$.

Example: The del Pezzo-1 surface (single blowup of $\mathbb{P}^2$) has two projective relations:

coords	powers Q^1	Q^2	divisor class
x_1	1	0	H
x_2	1	0	H
x_3	1	1	$H + X$
x_4	0	1	X

Divisors & line bundles

Line bundles $\mathcal{O}(D)$ and divisor D are in a direct correspondence:

$$\boxed{\text{divisor classes} \cong \text{line bundle classes}}$$

What now? 

Part 3

- An **algorithm** to compute line bundle-valued cohomology group dimensions

Remember: Ultimately, we are interested in computing $\dim H^i(\mathcal{X}; \mathcal{O}_{\mathcal{X}}(D))$.

[2×Blumenhagen-Jurke-Rahn-Roschy '10, Rahn-Roschy '10, Jow '10]

Input data: toric variety X

- homogeneous coordinates $H = \{x_1, \ldots, x_n\}$
- associated GLSM charges Q_i^a for each x_i
- Stanley-Reisner ideal $\mathrm{SR} = \langle \mathcal{S}_1, \ldots, \mathcal{S}_N \rangle$

Basic idea of the algorithm

Count monomials of a specific form which carry the same GLSM charge as the divisor D specifying the line bundle $\mathcal{O}_{\mathcal{X}}(D)$.

But: One also has to determine to which cohomology group dimension $h^i(X; \mathcal{O}_{\mathcal{X}}(D))$ (i.e. to which degree i) this counting contributes.

Take a squarefree monomial $\mathcal{Q} = x_{i_1} \cdots x_{i_k}$ of the coordinates H.

Consider monomials of the form

$$R^{\mathcal{Q}}(x_1, \ldots, x_n) = \frac{T(\overbrace{x_{j_1}, \ldots, x_{j_{n-k}}}^{\text{complement coordinates } H \setminus \mathcal{Q}})}{\underbrace{x_{i_1} \cdots x_{i_k}}_{\mathcal{Q}} \cdot W(\underbrace{x_{i_1}, \ldots, x_{i_k}}_{\text{coordinates in } \mathcal{Q}})} \qquad T,\ W \text{ monomials}$$

$$= \underbrace{x_{j_1}^{\rho_{j_1}} \cdots x_{j_{n-k}}^{\rho_{j_{n-k}}}}_{\text{positive powers } \geq 0} \cdot \underbrace{x_{i_1}^{-1-\rho_{i_1}} \cdots x_{i_k}^{-1-\rho_{i_k}}}_{\text{negative powers } \leq -1}$$

- Coordinates in $\mathcal{Q}$ have negative powers ≤ -1
- Remaining coordinates in $H \setminus \mathcal{Q}$ have positive powers ≥ 0

Step 1: Count number of monomials with degree of D

$$\boxed{\mathcal{N}_D(\mathcal{Q}) := \dim\left\{R^{\mathcal{Q}} : \deg_{\mathrm{GLSM}} R^{\mathcal{Q}} = D\right\}}$$

$\mathcal{Q} =$ squarefree monomial from the union of the coordinates in SR ideal generators

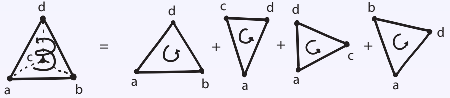
Determine to which cohomology group dimension $h^i(X;\mathcal{O}_X(D))$ the number $\mathcal{N}_D(\mathcal{Q})$ contributes.

➙ Trace back how often the same $\mathcal{Q}$ arises.

For each $\mathcal{Q}$ build up an abstract simplex $\Gamma^{\mathcal{Q}} := \{S \subset \mathrm{SR} : \mathcal{Q}(S) = \mathcal{Q}\}$ with k-faces

$$F_k(\Gamma^{\mathcal{Q}}) := \left\{S \in \Gamma^{\mathcal{Q}} : |S| = k+1\right\}.$$

$$\phi_k : F_k(\Gamma^{\mathcal{Q}}) \longrightarrow F_{k-1}(\Gamma^{\mathcal{Q}})$$
$$e_\rho \mapsto \sum_{s\in\rho} \mathrm{sign}(s,\rho) e_{\rho-\{s\}}$$



defines the boundary mappings, where $e_{\rho-\{s\}} = 0$ if $e_{\rho-\{s\}} \notin \Gamma^{\mathcal{Q}}$.

Step 2: Multiplicity factor and group contribution

Consider the (reduced) homology $\tilde{H}_\bullet(\Gamma^{\mathcal{Q}})$ and define the multiplicity factors

$$\boxed{\mathfrak{h}_i(\mathcal{Q}) := \dim \tilde{H}_{|\mathcal{Q}|-i-1}(\Gamma^{\mathcal{Q}})}$$

Those multiplicity factors are 0 or 1 in most cases—but not always.

"Dirty trick": Via exactness it often suffices to determine just $\dim F_k(\Gamma^{\mathcal{Q}})$.

Dimension of line bundle sheaf cohomology

$$\dim H^i(X;\mathcal{O}_X(D)) = \sum_{\mathcal{Q}} \underbrace{\mathfrak{h}_i(\mathcal{Q})}_{\text{multiplicity factor}} \cdot \overbrace{\mathcal{N}_D(\mathcal{Q})}^{\#(\text{suitable monomials } R^{\mathcal{Q}})}$$

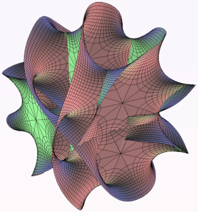
sum ranges over square-free monomials from unions of SR generators

1. Determine all monomials $\mathcal{Q}$ from unions of SR gens.
2. For each such $\mathcal{Q}$ compute the corresponding numbers of SR gen. combinations $F_k(\Gamma^{\mathcal{Q}})$
3. From those determine the multiplicity factors $\mathfrak{h}_i(\mathcal{Q})$
4. For each $\mathcal{Q}$ where $\mathfrak{h}_i(\mathcal{Q}) \neq 0$ count the number of rational functions $\mathcal{N}_D(\mathcal{Q})$.
5. Sum over all relevant contributions $\mathfrak{h}_i(\mathcal{Q}) \cdot \mathcal{N}_D(\mathcal{Q})$.

→ **completely algorithmic**

Part 4

- **Applications**
- Cohomology of subvarieties
- Tangent bundle of complete intersections
- Finite group actions (orbifolds & orientifolds)



[Blumenhagen-Jurke-Rahn-Roschy '10]

From the cohomology of a toric ambient space one can descent to the cohomology of a hypersurface, e.g. a Calabi-Yau hypersurface like $\mathbb{P}^4[5]$.

The Koszul sequence

Let S be a hypersurface (i.e. divisor) in a toric variety X and T be an arbitrary second divisor of X.

$$0 \longrightarrow \underbrace{\mathcal{O}_X(T-S) \hookrightarrow \mathcal{O}_X(T)}_{\substack{\text{line bundles on} \\ \text{ambient space } X}} \twoheadrightarrow \underbrace{\mathcal{O}_S(T)}_{\substack{\text{line bundle on} \\ \text{hypersurface } S\subset X}} \longrightarrow 0.$$

➙ Compute $H^i(S;\mathcal{O}_S(T))$ via induced long exact cohomology sequence.

The complete intersection $S = S_1 \cap \cdots \cap S_t$ of several hypersurfaces $S_i \subset X$ can be handled by iteration.

Tangent bundle of hypersurfaces

On a complete intersection $S = S_1 \cap \cdots \cap S_t$ the tangent bundle T_S can be described via two short exact sequences:

The split Euler sequence

$$0 \longrightarrow \mathcal{O}_S^{\oplus r} \hookrightarrow \bigoplus_{k=1}^{n} \overbrace{\mathcal{O}_S(D_k)}^{\text{coordinate divisors}} \twoheadrightarrow \mathcal{E}_S \longrightarrow 0$$

$$0 \longrightarrow T_S \hookrightarrow \mathcal{E}_S \twoheadrightarrow \bigoplus_{i=1}^{t} \underbrace{\mathcal{O}_S(S_i)}_{\text{hypersurface divisors}} \longrightarrow 0$$

Use the Koszul sequence to compute the cohomology dimensions $h^i(S; \mathcal{O}_S(T))$ from the cohomology of $\mathcal{X}$.

➙ Laborious, but in principle “straightforward”.

IV. Applications: Finite group actions

In orbifold or orientifold constructions one has a discrete finite symmetry acting on the space-time and considers the quotient space.

➙ Consider the invariant part of the cohomology.

Equivariant structure on line bundles

Let G be a finite group acting holomorphically on X. The group element action $g : X \longrightarrow X$ on the base space may be lifted to the bundle mapping $\phi_g : L \longrightarrow L$. If

$$\phi_g \circ \phi_h = \phi_{gh}$$

$$\begin{tikzcd} L \arrow[r, dashed, "\phi_g"] \arrow[d, "\pi"'] \arrow[dr, phantom, "\circlearrowleft"] & L \arrow[d, "\pi"] \\ X \arrow[r, "g"] & X \end{tikzcd}$$

this defines an equivariant structure.

This gives a splitting of the cohomology classes:

$$H^i(\mathcal{X}; \mathcal{O}_\mathcal{X}(D)) = H^i_{\text{inv}}(\mathcal{X}; \mathcal{O}_\mathcal{X}(D)) \oplus H^i_{\text{non-inv}}(\mathcal{X}; \mathcal{O}_\mathcal{X}(D)).$$

[Cvetic-García-Etxebarria-Halverson '10]

Finite group actions

The dimensions of those splittings can be computed by applying the uplifted G-action on the monomials counted in $\mathcal{N}_D(\mathcal{Q}) = \mathcal{N}_{\text{inv}} \oplus \mathcal{N}_{\text{non-inv}}$.

Quotient space cohomology

$$h^i(\mathcal{X}/G; \underbrace{\widetilde{\mathcal{O}_\mathcal{X}(D)}}_{\text{bundle over the quotient space}}) = h^i_{\text{inv}}(\mathcal{X}; \mathcal{O}_\mathcal{X}(D)) = \sum_{\mathcal{Q}} \underbrace{\mathfrak{h}_i(\mathcal{Q})}_{\text{multiplicity factor}} \cdot \overbrace{\mathcal{N}_{D,\text{inv}}(\mathcal{Q})}^{\#(\text{suitable } G\text{-invariant monomials})}$$

Note that the multiplicity factors $\mathfrak{h}_i(\mathcal{Q})$ remain unchanged!

➙ Rather simple to compute!

Example: $\mathbb{CP}^2/\mathbb{Z}_3$

Consider the following $\mathbb{Z}_3$-action on $\mathbb{P}^2$:

$$g_1 : (x_1, x_2, x_3) \mapsto (\alpha x_1, \alpha^2 x_2, x_3) \qquad \text{for } \alpha := \sqrt[3]{1} = \mathrm{e}^{\frac{2\pi \mathrm{i}}{3}}.$$

Due to the projective equivalence $(x_1, x_2, x_3) \sim (\lambda x_1, \lambda x_2, \lambda x_3)$ between the homogeneous coordinates x_i this base involution is equivalent to

$$g_2 : (x_1, x_2, x_3) \mapsto (x_1, \alpha x_2, \alpha^2 x_3)$$
$$g_3 : (x_1, x_2, x_3) \mapsto (\alpha^2 x_1, x_2, \alpha x_3),$$

The same coordinates form the monomials $R^{\mathcal{Q}}$ used in the algorithm.

➙ Involution mapping can be applied to the monomials.

Choose g_1 to be the equivariant structure. For fixpoint-free actions (yielding smooth quotients) all equivariant structures are equivalent.

Example: $\mathbb{CP}^2/\mathbb{Z}_3$

Apply action to monomials that contribute to the cohomology and read off the corresponding parts by their phases.

Example: Monomials for $\mathcal{O}(-6)$ using g_1, i.e. $(x_1, \alpha x_2, \alpha^2 x_3)$:

$$\underbrace{\frac{1}{\alpha^4 u_1^4 \alpha^2 u_2 u_3}}_{g_1 \to 1}, \quad \underbrace{\frac{1}{\alpha u_1 \alpha^8 u_2^4 u_3}}_{g_1 \to 1}, \quad \underbrace{\frac{1}{\alpha u_1 \alpha^2 u_2 u_3^4}}_{g_1 \to 1}, \quad \underbrace{\frac{1}{\alpha^3 u_1^3 \alpha^4 u_2^2 u_3}}_{g_1 \to \alpha}, \quad \underbrace{\frac{1}{\alpha^3 u_1^3 \alpha u_2 u_3^2}}_{g_1 \to \alpha^2},$$

$$\underbrace{\frac{1}{\alpha^2 u_1^2 \alpha^6 u_2^3 u_3}}_{g_1 \to \alpha^2}, \quad \underbrace{\frac{1}{\alpha u_1 \alpha^6 u_2^3 u_3^2}}_{g_1 \to \alpha}, \quad \underbrace{\frac{1}{\alpha^2 u_1^2 \alpha^2 u_2 u_3^3}}_{g_1 \to \alpha}, \quad \underbrace{\frac{1}{\alpha u_1 \alpha^4 u_2^2 u_3^3}}_{g_1 \to \alpha^2}, \quad \underbrace{\frac{1}{\alpha^2 u_1^2 \alpha^4 u_2^2 u_3^2}}_{g_1 \to 1},$$

Result:

$$h^2(\mathbb{P}^2; \mathcal{O}(-6)) = (4_{\text{inv}}, 3_\alpha, 3_{\alpha^2}) \quad \rightarrow \quad h^2(\mathbb{P}^2/\mathbb{Z}_3; \widetilde{\mathcal{O}(-6)}) = 4$$

→ **Now: Make it simple & easy to use!**

Part 5

- Algorithm implementation **cohomCalg**
- Outlook

V. Implementation: **cohomCalg**

→ Google for **cohomCalg**

or try the core algorithm online:

→ `cohomcalg.benjaminjurke.net`

cohomCalg

high-speed, cross-platform
C++ implementation **cohomCalg**

- Windows / Mac / Linux
- open source, GLPv3
- multi-core support

cohomCalg Koszul extension

Mathematica interface

- Hypersurfaces & complete intersections
- (co-)tangent bundle, $\Lambda^2 T^* S$
- Hodge diamond
- Monads

Example: Line bundles on toric variety dP_3

```
Open Files
dP3.in

% The vertices and GLSM charges:
    vertex u1 | GLSM:  ( 1,  0,  0,  1 );
    vertex u2 | GLSM:  ( 1,  0,  1,  0 );
    vertex u3 | GLSM:  ( 1,  1,  0,  0 );
    vertex u4 | GLSM:  ( 0,  0,  0,  1 );
    vertex u5 | GLSM:  ( 0,  0,  1,  0 );
    vertex u6 | GLSM:  ( 0,  1,  0,  0 );

% The Stanley-Reisner ideal:
    srideal [u1*u2,  u1*u3,  u1*u4,  u2*u3,
             u2*u5,  u3*u6,  u4*u5,  u4*u6,  u5*u6];

% And finally the requested line bundle cohomologies:
    ambientcohom O( -2,  0, -2,  0 );
    ambientcohom O( -3,  2, -2, -1 );
```

```
Administrator: Command Prompt

D:\package\bin>cohomcalg --hideinput --nomonomfile dP3.in

    cohomCalg v0.31
    (compiled on May 25 2011 @ 10:43:46 for Windows x86-64 / 64 bit)
    author: Benjamin Jurke (mail@bjurke.net)
    Koszul extension: Thorsten Rahn (thorsten.rahn@gmail.com)
    Based on the algorithm presented in arXiv:1003.5217

Reading in the input file 'dP3.in'...


Usage and generation of intermediate monomial files deactivated.

Starting computation of secondary sequences...

Computation of secondary cohomologies and contributions complete.
Computation of the target cohomology group dimensions complete.

Cohomology dimensions:
======================
    dim H^i(A; O( -2,  0, -2,  0 )) = (  0,  2,  0 )

    dim H^i(A; O( -3,  2, -2, -1 )) = (  0, 10,  0 )


    All done. Programm run successfully completed.
```

The **C++ core program** takes care of the actual algorithm that computes line bundles on toric spaces.

Example: The resolved $\mathbb{P}_{11222}[8]$

```
In[181]:= P11222BlowUp = {
              (*Coordinates*){v1, v2, v3, v4, v5, vX},
              (*Stanley Reisner*){{v1, v2}, {v3, v4, v5, vX}},
              (*Equivalence Relations*){{1, 0}, {1, 0}, {2, 1}, {2, 1}, {2, 1}, {0, 1}}
          };
          CalabiYauHyperSurface = {{8, 4}};
          CohomologyOf["TangentBundle", P11222BlowUp, CalabiYauHyperSurface, "Calabi-Yau", "Verbose2"]
```

$\mathcal{O}_S^{r\,\oplus}$	$\oplus_{k=1}^{n}[\mathcal{O}_S[D_k]]$ $\mathcal{O}_S[1,0]^{\oplus 2} \oplus \mathcal{O}_S[2,1]^{\oplus 3} \oplus \mathcal{O}_S[0,1]$	$\mathcal{E}_S$
2	23	A_{21}
0	3	A_{22}
0	0	A_{23}
2	0	A_{24}
0	0	0

$\Longrightarrow$

$\mathcal{E}_S$
21
3
2
0
0

T_S	$\mathcal{E}_S$	$\oplus_{i=1}^{1}[\mathcal{O}_S[S_i]]$ $\mathcal{O}_S[8,4]$
0	21	104
A_{32}	3	0
A_{33}	2	0
0	0	0
0	0	0

$\Longrightarrow$

T_S
0
86
2
0
0

```
Calculation of TangentBundle cohomology done:
--------------------------------------------------
Total number of line bundles:   42
Newly computed:                 18
Total time needed:              0.593 seconds
```

$h^\bullet[T_S]$
0
86
2
0

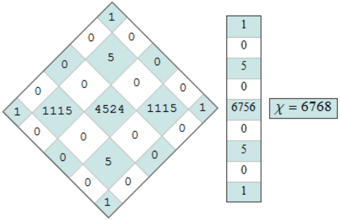
cohomCalg ++Koszul extension

CICY 4-fold in the context of F-Theory GUT Vacua

```
In[53]:= Example4Fold = {
    (*Coordinates*){v1, v2, v3, v4, v5, v6, v1s, v7, v8, v9, v10},
    (*Stanley Reisner*){{v3, v9}, {v5, v9}, {v7, v10}, {v1, v2, v3}, {v4, v1s, v8}, {v4, v7, v8}, {v4, v8, v9},
                        {v5, v6, v1s}, {v5, v6, v10}, {v1, v2, v6, v1s}},
    (*Equivalence Relations*){{3, 3, 3, 3, 0}, {2, 2, 2, 2, 0}, {1, 0, 0, 0, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 1, 0},
                        {0, 1, 0, 0, 0}, {0, 1, 1, 0, 0}, {0, 0, 1, 0, 1}, {0, 0, 1, 0, 0}, {0, -1, -1, 1, -1}, {0, 0, 0, 0, 1}}};
CompleteIntersection = {{6, 6, 6, 6, 0}, {0, 0, 2, 1, 1}};

CohomologyOf["HodgeDiamond", Example4Fold, CompleteIntersection, "Calabi-Yau"]
```

```
Calculation of HodgeDiamond cohomology done:
---------------------------------------------
Total number of line bundles:   478
Newly computed:                 264
Total time needed:              31.233 seconds
```



[Grimm-Krause-Weigand '09]

The **Mathematica frontend** provides convenient functionality, utilizing the previously discussed methods.

Conclusions

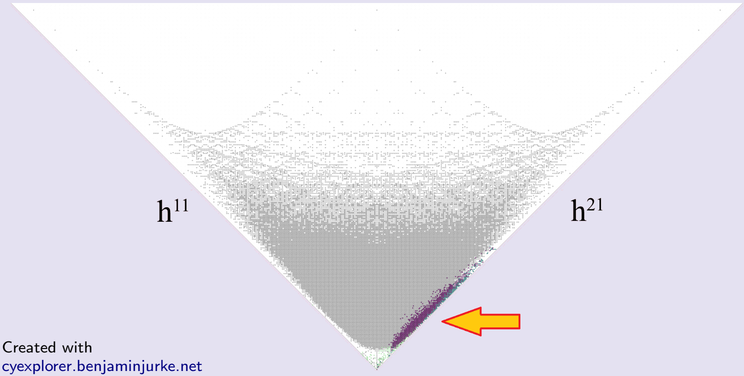
Presented material

- An easy and efficient way to compute line bundle-valued cohomology group dimensions of a toric variety.
- The algorithm implementation **cohomCalg**.
- Methods for the computation of various vector bundles on toric subspaces with a convenient Mathematica frontend.
- An extension of the proposed algorithm to compute quotient space cohomology of a toric variety, i.e. methods to calculate line bundle cohomology on orbifolds and orientifolds.

Outlook

Related work in progress & ideas

- Exploration of target space duality in $(0, 2)$ het. models. [Blumenhagen-Rahn '11 (upcoming)]
- Construction and analysis of new Calabi-Yau 3-folds. [Jurke-Rahn '11 (upcoming)]



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Outlook

Long term applications

- Scans over extremely large string landscape sets and analysis of various phenomenological properties...

  ➙ Cloud computing!

The end

Outlook

Long term applications

- Scans over extremely large string landscape sets and analysis of various phenomenological properties...

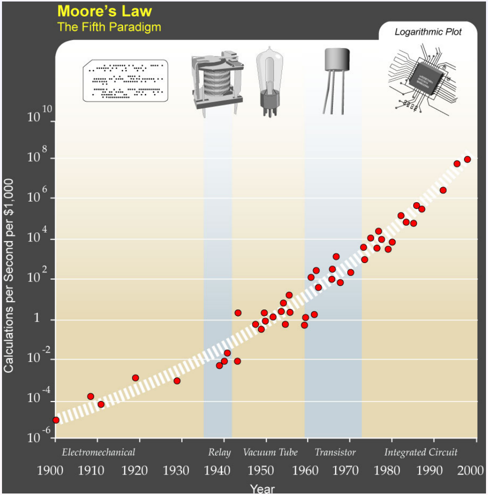
  → Cloud computing!

The end

...well, not quite yet!

Part 6

- What is “the Cloud”?
- How can it be utilized for science?



→ Over time: "More bang for the buck!"

What is “the Cloud”?

PDP-10 ➙ PC ➙ ?

- 1966: PDP-10 — First computer that made time-sharing common!
  ➙ Small terminals accessing huge mainframe
- 1981: IBM PC — First computer for personal / private home use!
  ➙ Small independant versatile powerhouses
- Late 2000s: More and more services remotely accessed.
  ➙ A step back?

The improvements in computer speed allow to raise the level of abstraction by a huge margin. Classical approaches like time sharing are replaced by true virtualization.

Virtualization

A virtual machine simulates all relevant structures of a computer. From a programmers perspective one effectively operates on an entirely seperate computer.

➙ One no longer has to care about the hardware details of the machine!

On modern machines the performance losses of virtualization for most applications are no longer significant.

➙ True detachment of hardware and software!

Remote (Super-)Computing — Pros

Using virtualization one can effectively rent a remote computer and for all practical purposes operate on it similar to a local machine.

- Maintenance: One big centralized computer cluster is much easier to maintain than thousands of smaller systems.
- Load spikes & uptime: Since a virtual machine is not bound to specific hardware, one can easily move it to a different machine or (dynamically) associate more hardware to it.
- Costs: Due to dynamic allocation of resources, the actual hardware is more efficiently used.

Remote (Super-)Computing — Cons

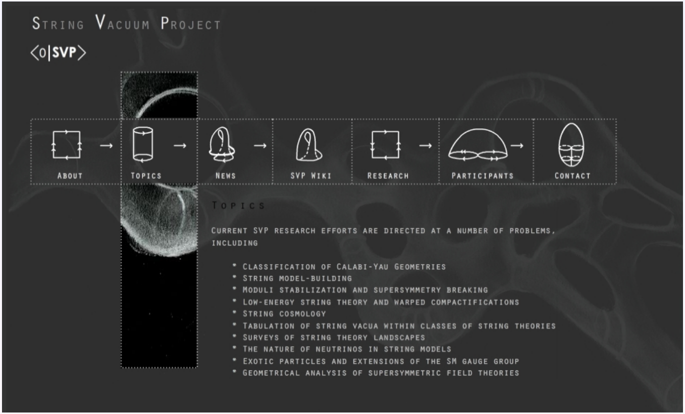
However, there are also new challenges to be faced:

- Restrictions: The virtualization may be limited to certain specific (virtual) operating systems, which makes it challenging (or impossible) to use existing software.
- Security: The data is no longer “at home”, privacy questions arise.

$\downarrow$

Do those limitations largely affect scientific computing?

VI. Cloud Computing: String Vacuum Project



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