Scanning for SWISS CHEESE geometries

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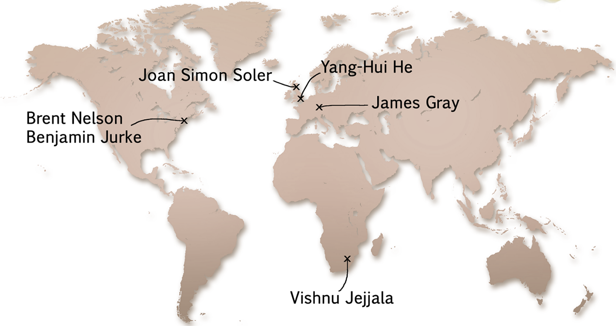
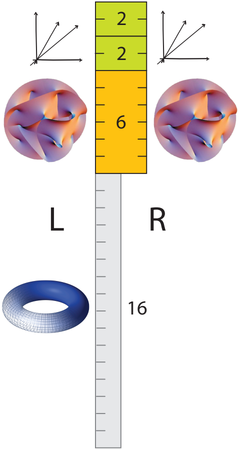
Outline

1. The Large Volume Scenario
2. Swiss Cheese reformulated
3. Scan Implementation

Collaboration:



The Issue of Moduli Stabilization



To make contact with the observed world, 10d string theory needs to be compactified down to 4d.

→ **Various choices for 6d internal space** (topology, curvature, complex structure, ...)

Most common:
Compact Calabi-Yau threefolds
(complex Ricci-flat Kähler manifolds with $SU(3)$ holonomy/structure group)

Focus here: type IIB string theory

Unstabilized Moduli?

Calabi-Yau geometries have $h^{1,1}$ Kähler moduli and $h^{2,1}$ complex structure moduli. Plus there is the type-IIB axio-dilaton modulus S.

What happens to unstabilized moduli?
Compactification leaves effective 4d $\mathcal{N}=1$ supersymmetric theory.
Moduli fields appear as the massless scalar bosons in chiral superfields.

➙ Left "unstabilized" those massless superfields have no potential and would lead to 5th force effects or missing energy in colliders.

Not observed in nature! ➙ **Moduli must be stabilized!**

↓

General idea:
Use fluxes to generate a potential and a nonzero vev for moduli.

Effective Type IIB theory

The IIB theory's effective 4d $\mathcal{N}=1$ **Kähler potential** takes the form

$$K = -\log(S+\bar{S}) - \log\left(-\mathrm{i}\int_{\mathcal{X}} \Omega_3 \wedge \bar{\Omega}_3\right) - 2\log\left(\hat{\mathcal{V}} + \frac{\xi}{2g_s^{\frac{3}{2}}}\right)$$

where $(\alpha')^3$ string (but no loop!) corrections enter via the ξ, i.e.

$$\xi = -\chi(\mathcal{X})\frac{\zeta(3)}{2(2\pi)^3}$$

Important: If the α' corrections are ignored (letting $\xi \to 0$), the associated **Kähler metric $\boldsymbol{K_{\alpha\beta} = \partial_\alpha \partial_\beta K}$** is block-diagonal with respect to the three moduli types U^k, T_i and S.

➙ Assumed in the **KKLT moduli stabilization scenario**.

The KKLT Scenario

Using fluxes $F_3 = \mathrm{d}C_2$ and $H_3 = \mathrm{d}B_2$, consider the

Gukov-Vafa-Witten superpotential:

$$W_{\mathrm{GVW}} = \int_{\mathcal{X}} \Omega_3 \wedge G_3, \quad \underbrace{G_3 = \bar{F}_3 - \mathrm{i}S\bar{H}_3}_{G_3\text{-flux}}$$

$\downarrow$ F-terms

The **associated F-term potential V_F** can be brought into the form

$$V_F = \mathrm{e}^K \Big(K^{-1}_{U\bar{U}} F_U \bar{F}_{\bar{U}} + K^{-1}_{S\bar{S}} F_S \bar{F}_{\bar{S}} \Big).$$

which does not depend on the Kähler moduli (cancellation due to the block-diagonal form of the (inverse) Kähler metric since α' corrections are neclected).

[Kachru-Kallosh-Linde-Trivedi 2003]

The KKLT Scenario

Minimizing V_F therefore only fixes the complex structure moduli and axio-dilaton. What about the Kähler moduli?

In addition to W_{GVW}, the superpotential W also involves **non-perturbative contributions** from e.g. E3-brane instantons:

$$W_{\text{np}} = \sum_{\rho} A_{\rho}(U^k, S) \mathrm{e}^{-a_\rho T_\rho} \quad \text{where} \quad T_\rho = \sum_{i=1}^{h^{1,1}(\mathcal{X})} m_\rho^i T_i$$

Here $A_\rho(U^k, S)$ are treshold prefactors and
the coefficients a_ρ determine the type of the contribution.

Define $W_0 = W_{\text{GVW}}|_{\text{min}}$ to be the minimum of the GVW superpotential for fixed complex structure moduli and axio-dilaton.

The KKLT Scenario

Consider the **full superpotential** $W = W_0 + \sum_\rho A_\rho(U^k, S)\mathrm{e}^{-a_\rho T_\rho}$.

$$\downarrow$$

associated F-term potential: $V_F = \mathrm{e}^K \left(K^{-1}_{T\bar{T}} F_T \bar{F}_{\bar{T}} - 3|W|^2 \right)$
Minimizing this potential stabilizes the Kähler moduli as well.

So, in KKLT we have a **two-step moduli stabilization**:

1. Stabilize the complex structure and axio-dilaton by the Gukov-Vafa-Witten superpotential, i.e. via G_3-flux.
2. Then consider the non-perturbative contributions as a perturbation around this minimum and stabilize the Kähler structure.

$$\downarrow$$

Rather artificial... can we improve this?

The LARGE Volume Scenario

Idea: Consider the $\boldsymbol{\mathcal{V}}$**-dependency** of the KKLT approach.

By taking the overall volume $\mathcal{V}$ large, we have $\frac{1}{\mathcal{V}} \ll 1$ such that we can expand in the inverse volume. We are now taking the full superpotential:

$$W = \int_{\mathcal{X}} \Omega_3 \wedge G_3 + \sum_{\rho} A_{\rho}(U^k) \mathrm{e}^{-a_{\rho} T_{\rho}}$$

$$\downarrow$$

The expansion of the V_F contributions in $\frac{1}{\mathcal{V}}$ shows the following:

- $K^{-1}_{U\bar{U}} F_U \bar{F}_{\bar{U}} \sim \mathcal{O}(\frac{1}{\mathcal{V}^0})$
- $K^{-1}_{S\bar{S}} F_S \bar{F}_{\bar{S}} \sim \mathcal{O}(\frac{1}{\mathcal{V}^0})$
- $K^{-1}_{S\bar{T}} F_S \bar{F}_{\bar{T}} \sim \mathcal{O}(\frac{1}{\mathcal{V}^1})$
- $K^{-1}_{T\bar{T}} F_T \bar{F}_{\bar{T}} - 3|W|^2 \sim \mathcal{O}(\frac{1}{\mathcal{V}^1})$

The LARGE Volume Scenario

To leading order we then obtain the F-term potential:

$$V_F = \mathrm{e}^K \left(K^{-1}_{U\bar{U}} F_U \bar{F}_{\bar{U}} + K^{-1}_{S\bar{S}} F_S \bar{F}_{\bar{S}} \right) + \mathcal{O}(\tfrac{1}{\mathcal{V}^3})$$

$$\downarrow$$

Same potential as in the 1st KKLT step!
(and only the W_{GVW} part of W contributes)

Once again use $W_0 = W_{\mathrm{GVW}}|_{\mathrm{min}}$ to stabilize U^k and S. The $\mathcal{O}(\frac{1}{\mathcal{V}^3})$ term of V_F as a perturbation then stabilizes the T_i moduli.

➙ The benefit of the LARGE Volume Scenario is therefore a **$\mathcal{V}$-controlled separation of the two moduli stabilization steps**.

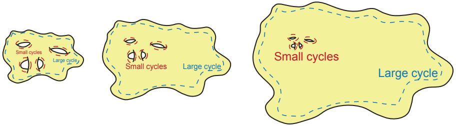
Geometric prerequisites

In order for the LARGE Volume Scenario to work out, we have a number of **requirements for the geometry of the Calabi-Yau**:

- We require a number of (small) 4-cycles that can be wrapped by E3-branes to generate the non-perturbative contributions for the 2nd Kähler moduli stabilization step.
- At the same time, the overall geometry $\mathcal{V}$ must be able to scale to large values to allow for the 2-step approach. This requires at least one (large) 4-cycle that can be scaled to large values.

$$\Downarrow$$

$$\mathcal{V} \sim \tau_{\mathrm{L}}^{\frac{3}{2}} - \sum_{i=1}^{N_{\mathrm{small}}} \tau_{\mathrm{s}_i}^{\frac{3}{2}}$$



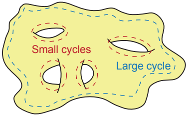
Swiss Cheese geometry

The LARGE Volume Scenario Claim

$\mathcal{X}$ Calabi-Yau threefold, $D_1, \dots, D_n \subset \mathcal{X}$ divisors, $\tau_i := \mathrm{vol}(D_i)$

LARGE Volume Scenario Claim: Let the limit be taken as

LV Limit: $\begin{cases} \tau_1, \dots, \tau_{N_{\text{small}}} \text{ remain small} \\ \mathcal{V} \to \infty \text{ for } \tau_{N_{\text{small}}+1}, \dots, \tau_{h^{1,1}(\mathcal{X})} \to \infty \end{cases}$



such that the Kähler potential K and the superpotential W in type IIB $\mathcal{N}=1$ 4d SUGRA

$$K = \langle K_{\text{cs}} \rangle - 2 \ln \left(\hat{\mathcal{V}} + \hat{\xi} \right), \quad W = \langle W_{\text{GVW}} \rangle + \sum_{i=1}^{N_{\text{small}}} A_i(S, U_j) \mathrm{e}^{-a_i T_i}$$

Then the scalar potential V admits a set of AdS non-SUSY minima.

[Cicoli-Conlon-Quevedo 2008]

The Goal

Not too many geometries known...

$\downarrow$

Develop an **algorithmic approach to identify the Swiss Cheese property** in a given Calabi-Yau threefold.

Step 0: What do we have?

Formulation in the LVS Claim:

$$\text{LV Limit:} \quad \begin{cases} \tau_1, \ldots, \tau_{N_{\text{small}}} \text{ remain small} \\ \mathcal{V} \to \infty \text{ for } \tau_{N_{\text{small}}+1}, \ldots, \tau_{h^{1,1}(\mathcal{X})} \to \infty \end{cases}$$

$$\Downarrow$$

Implied starting assumption: We are in a *"convenient"* basis!
...means that the divisor basis can be directly split up into

$$D_1, \ldots, D_n = \underbrace{D_{\mathrm{L}_1}, \ldots, D_{\mathrm{L}_{n_\mathrm{L}}}}_{\text{large cycle divisors}}, \underbrace{D_{\mathrm{s}_1}, \ldots, D_{\mathrm{s}_{n_\mathrm{s}}}}_{\text{small cycle divisors}}$$

➙ Computationally, that's the most oversimplifying assumption ever!
"Manifest Swiss Cheese"

Step 1: Reformulate Large Volume Limit

$$\text{4-cycle volumes } \tau_i \xleftrightarrow{\text{Poincaré}} \text{Kähler parameters } t^i$$

Central idea: Rewrite LVL in terms of Kähler parameters t^i!

intersection form matrix on divisor D_i: $\quad (\kappa_{(i)})_{jk} := \kappa_{ijk}$

divisor 4-cycle volume: $\quad \tau_i = \frac{1}{2}\kappa_{ijk}t^j t^k = \frac{1}{2}\vec{t}^{\,*}\kappa_{(i)}\vec{t}$

split Kähler parameter vector: $\quad \vec{t} = \underbrace{\sum_{A=1}^{n_{\mathrm{L}}} \lambda_A \vec{t}_{\mathrm{L}_A}}_{\text{large}} + \underbrace{\sum_{a=1}^{n_{\mathrm{s}}} \gamma_a \vec{t}_{\mathrm{s}_a}}_{\text{small}} \in \mathcal{K}$

$\downarrow$

In the LVL claim $\mathcal{V} \to \infty$ where $\tau_{N_{\mathrm{small}}+1}, \ldots, \tau_{h^{1,1}(\mathcal{X})} \to \infty$ then corresponds to $\lambda_A \to \infty$.

Step 1: Reformulate Large Volume Limit

What does small and large mean in terms of the Kähler parameters?

$$\tau_i = \frac{1}{2}\Big[\overbrace{\lambda_A\lambda_B \cdot (\vec{t}^{\,*}_{\mathrm{L}_A}\kappa_{(i)}\vec{t}_{\mathrm{L}_B}) + 2\lambda_A\gamma_b \cdot (\vec{t}^{\,*}_{\mathrm{L}_A}\kappa_{(i)}\vec{t}_{\mathrm{s}_b})}^{\text{terms involving large parameters } \lambda_A} + \gamma_a\gamma_b \cdot (\vec{t}^{\,*}_{\mathrm{s}_a}\kappa_{(i)}\vec{t}_{\mathrm{s}_b})\Big]$$

$$\downarrow$$

large 4-cycles τ_I: $\quad \vec{t}^{\,*}_{\mathrm{L}_A}\kappa_{(I)}\vec{t}_{\mathrm{L}_B} \neq 0 \quad \text{OR} \quad \vec{t}^{\,*}_{\mathrm{L}_A}\kappa_{(I)}\vec{t}_{\mathrm{s}_b} \neq 0$

small 4-cycles τ_α: $\quad \underbrace{\vec{t}^{\,*}_{\mathrm{L}_A}\kappa_{(\alpha)}\vec{t}_{\mathrm{L}_B} = 0 \quad \text{AND} \quad \vec{t}^{\,*}_{\mathrm{L}_A}\kappa_{(\alpha)}\vec{t}_{\mathrm{s}_b} = 0}_{\kappa_{(\alpha)}\vec{t}_{\mathrm{L}_A} = 0}$

Step 1: Reformulate Large Volume Limit

Blowup mode condition of the inverse Kähler metric: $K^{-1}_{\alpha\alpha} \sim \mathcal{V}\sqrt{\tau_\alpha}$
For a general Calabi-Yau manifold there is a K^{-1}_{ij} expansion

$$\begin{aligned} K^{-1}_{ij} &= -\frac{2}{9}\left(2\mathcal{V} + \hat{\xi}\right)\kappa_{ijk}t^k + \frac{4\mathcal{V} - \hat{\xi}}{\mathcal{V} - \hat{\xi}}\tau_i\tau_j \\ &= -\frac{4}{9}\mathcal{V}\kappa_{ijk}t^k + 4\tau_i\tau_j + \mathcal{O}\left(\tfrac{1}{\mathcal{V}^1}\right) \end{aligned}$$

$$\Downarrow$$

$$\frac{K^{-1}_{\alpha\alpha}}{\mathcal{V}} \approx -\frac{4}{9}\kappa_{\alpha\alpha i}t^i = -\frac{4}{9}(\kappa_{(\alpha)}\vec{t})_\alpha \sim \sqrt{\tau_\alpha} = \sqrt{\vec{t}^*\kappa_{(\alpha)}\vec{t}}$$

$\kappa_{(\alpha)}\vec{t}_{\mathrm{L}_A} = 0$ ➞ RHS only depends on small cycle volumes $\vec{t}_{\mathrm{s}_\alpha}$.

➞ requires $(\kappa_{(\alpha)}\vec{t}_{\mathrm{s}_a})_\alpha \neq 0$ for at least one a

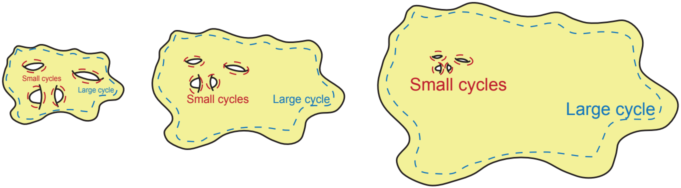
Step 1: Reformulate Large Volume Limit

Furthermore, the $\vec{t}_{\mathrm{L}_A}$ and $\vec{t}_{\mathrm{s}_a}$ have to be a basis (due to Poincaré)

$$\det\left(\vec{t}_{\mathrm{L}_1}, \ldots, \vec{t}_{\mathrm{L}_{N_{\mathrm{large}}}}, \vec{t}_{\mathrm{s}_1}, \ldots, \vec{t}_{\mathrm{s}_{N_{\mathrm{small}}}}\right) \neq 0,$$

which automatically takes care of the large cycles.
Also $\vec{t}$ has to be in the Kähler cone $\mathcal{K}$

$$\mathcal{K}^{\rho}{}_{i}\left(\lambda_A(\vec{t}_{\mathrm{L}_A})^i + \gamma_a(\vec{t}_{\mathrm{s}_a})^i\right) > 0$$



Step 1: Reformulated Large Volume Limit

With respect to a "**convenient**" basis we ultimately need to test if

$$\begin{cases} \text{small cycles:} & \kappa_{(\alpha)}\vec{t}_{\mathrm{L}_A} = 0 \\ K^{-1}_{\alpha\alpha} \text{ condition:} & (\kappa_{(\alpha)}\vec{t}_{\mathrm{s}_a})_\alpha \neq 0 \\ \text{non-triviality:} & \det(\vec{t}_{\mathrm{L}_1}, \ldots, \vec{t}_{\mathrm{L}_{N_{\mathrm{large}}}}, \vec{t}_{\mathrm{s}_1}, \ldots, \vec{t}_{\mathrm{s}_{N_{\mathrm{small}}}}) \neq 0 \\ \text{Kähler cone:} & \mathcal{K}^\rho{}_i\big(\lambda_A(\vec{t}_{\mathrm{L}_A})^i + \gamma_a(\vec{t}_{\mathrm{s}_a})^i\big) > 0 \end{cases}$$

has a solution, solving for all $\vec{t}_{\mathrm{L}_A}$, $\vec{t}_{\mathrm{s}_a}$, λ_A and γ_a.

Note that in this equation system only the $\kappa_{(i)}$s are coordinate-dependant.

Step 2: Arbitrary basis considerations

Life is hard... and most bases are rather inconvenient...

Let $\tilde{D}_{\tilde{1}}, \ldots, \tilde{D}_{\tilde{n}}$ be a **generic basis** and $A_i{}^{\tilde{\jmath}}$ a base change matrix relating to the convenient basis $D_1, \ldots, D_n$.

$$\begin{cases} \text{small cycles:} & A_\alpha{}^{\tilde{\jmath}} \kappa_{(\tilde{\jmath})} \vec{t}_{\mathrm{L}_A} = 0 \\ K^{-1}_{\alpha\alpha} \text{ condition:} & A_\alpha{}^{\tilde{\imath}} A_\alpha{}^{\tilde{\jmath}} (\kappa_{(\tilde{\imath})} \vec{t}_{\mathrm{s}_a})_{\tilde{\jmath}} \neq 0 \\ \text{non-triviality:} & \det(\vec{t}_{\mathrm{L}_1}, \ldots, \vec{t}_{\mathrm{L}_{N_{\mathrm{large}}}}, \vec{t}_{\mathrm{s}_1}, \ldots, \vec{t}_{\mathrm{s}_{N_{\mathrm{small}}}}) \neq 0 \\ \text{Kähler cone:} & \text{in a moment...} \\ \text{base change:} & \det(A) \neq 0 \end{cases}$$

Step -1: Kähler cone normalization

general Kähler cone condition: $\sum_{i=1}^{h^{1,1}} \mathcal{K}^{\kappa}{}_{i} t^{i} \geq 0$ for $\kappa = 1, \ldots, n_F$

Kähler cone **simplicial** ➞ $n_F = h^{1,1}$ ➞ $\mathcal{K}$ invertible matrix

Transform intersection form and Kähler cone to standard form via

$$\hat{D}_{\hat{\imath}} = \sum_{i} \hat{D}_{i} (\mathcal{K}^{-1})^{i}{}_{\hat{\imath}},$$

$$\hat{t}^{\hat{\imath}} = \sum_{i} \mathcal{K}^{\hat{\imath}}{}_{i} t^{i},$$

$$\hat{\kappa}_{\hat{\imath}\hat{\jmath}\hat{k}} = \sum_{i,j,k} \kappa_{ijk} (\mathcal{K}^{-1})^{i}{}_{\hat{\imath}} (\mathcal{K}^{-1})^{j}{}_{\hat{\jmath}} (\hat{\mathcal{K}}^{-1})^{k}{}_{\hat{k}}.$$

➞ normalized Kähler cone: $\hat{t}^{\hat{\imath}} > 0$

Step 3: Final Swiss Cheese detector

Let $\tilde{D}_{\tilde{1}}, \ldots, \tilde{D}_{\tilde{n}}$ be a **generic basis** with a **normalized Kähler cone**.

$$\begin{cases} \text{small cycles:} & {A_\alpha}^{\tilde{\jmath}} \kappa_{(\tilde{\jmath})} \vec{t}_{\mathrm{L}_A} = 0 \\ K^{-1}_{\alpha\alpha} \text{ condition:} & {A_\alpha}^{\tilde{\imath}} {A_\alpha}^{\tilde{\jmath}} (\kappa_{(\tilde{\imath})} \vec{t}_{\mathrm{s}_a})_{\tilde{\jmath}} \neq 0 \\ \text{non-triviality:} & \det(\vec{t}_{\mathrm{L}_1}, \ldots, \vec{t}_{\mathrm{L}_{N_{\mathrm{large}}}}, \vec{t}_{\mathrm{s}_1}, \ldots, \vec{t}_{\mathrm{s}_{N_{\mathrm{small}}}}) \neq 0 \\ \text{Kähler cone:} & \lambda_A (\vec{t}_{\mathrm{L}_A})^{\tilde{\imath}} + \gamma_a (\vec{t}_{\mathrm{s}_a})^{\tilde{\imath}} > 0 \\ \text{base change:} & \det(A) \neq 0 \end{cases}$$

$\downarrow$

Check the solvability of the system for A, $\vec{t}_{\mathrm{L}_A}$, $\vec{t}_{\mathrm{s}_a}$, λ_A and γ_a over $\mathbb{R}$.

→ Still a demanding task — $\boldsymbol{2(h^{1,1})^2 + h^{1,1}}$ **variables** — but doable!

Redundancy fixings

- $h^{1,1} = 2$: just implement the equation system in Mathematica and brute-force `FindInstance` it.
- $h^{1,1} \geq 3$: the inequality solver in Mathematica is way too slow.

$$\Downarrow$$

Using redundancies in the variables, turn inequalities into equalities:

$$\begin{cases} \text{small cycles:} & {A_\alpha}^{\tilde{\jmath}} \kappa_{(\tilde{\jmath})} \vec{t}_{\mathrm{L}_A} = 0 \\ K^{-1}_{\alpha\alpha} \text{ condition:} & {A_\alpha}^{\tilde{\imath}} {A_\alpha}^{\tilde{\jmath}} (\kappa_{(\tilde{\imath})} \vec{t}_{\mathrm{s}_a})_{\tilde{\jmath}} \neq 0 \\ \text{non-triviality:} & \det(\vec{t}_{\mathrm{L}_1}, \ldots, \vec{t}_{\mathrm{L}_{N_\text{large}}}, \vec{t}_{\mathrm{s}_1}, \ldots, \vec{t}_{\mathrm{s}_{N_\text{small}}}) = \pm 1 \\ \text{Kähler cone:} & \lambda_A (\vec{t}_{\mathrm{L}_A})^{\tilde{\imath}} + \gamma_a (\vec{t}_{\mathrm{s}_a})^{\tilde{\imath}} > 0 \\ \text{base change:} & \det(A) = \begin{cases} 1 & \text{for } h^{1,1} \text{ even} \\ \pm 1 & \text{for } h^{1,1} \text{ odd} \end{cases} \end{cases}$$

Actual implementation & application

- The $K^{-1}_{\alpha\alpha}$ condition is very non-restrictive and almost never rules out a model for $h^{1,1} > 2$ ➙ ignore it at first.
- In the end Mathematica is simply too slow. Along comes Singular!

➙

- Using further redundancies, numerous components of the matrix A and the vectors $\vec{t}_{\mathrm{L}_A}$, $\vec{t}_{\mathrm{s}_a}$ can be fixed, reducing the number of variables.

↓

Identifying the right combination of “trickery” was the main effort...

Actual implementation & application

Final implementation of the **Swiss Cheese test:**

1. Fix number of large / small cycles to test.
2. Compute dimension of Gröbner basis of the equation system

$$\begin{cases} \text{small cycles:} & A_{\alpha}{}^{\tilde{\jmath}}\kappa_{(\tilde{\jmath})}\vec{t}_{\mathrm{L}_A} = 0 \\ \text{non-triviality:} & \det(\vec{t}_{\mathrm{L}_1},\ldots,\vec{t}_{\mathrm{L}_{N_{\mathrm{large}}}},\vec{t}_{\mathrm{s}_1},\ldots,\vec{t}_{\mathrm{s}_{N_{\mathrm{small}}}}) = \pm 1 \\ \text{base change:} & \det(A) = \begin{cases} 1 & \text{for } h^{1,1} \text{ even} \\ \pm 1 & \text{for } h^{1,1} \text{ odd} \end{cases} \end{cases}$$

3. If non-negative, perform primary decomposition of Gröbner basis
4. Add Kähler cone condition and attempt to find solution over $\mathbb{R}$ using Mathematica in at least one component.
5. Check $K^{-1}_{\alpha\alpha}$ condition from the result. ➙ **Swiss Cheese**

Outlook

So far & currently:

- Scanned $h^{1,1} \leq 4$ for $n_{\text{large}} = 1$, but test generalizes to $n_{\text{large}} > 1$

	$h^{1,1} = 2$	$h^{1,1} = 3$	$h^{1,1} = 4$
# polytopes:	36	244	1197
# triangulations:	39	306	5930
# simplicial Kähler cones:	39	266	3513
# Swiss Cheese mflds:	**22**	**93**	**302**

- Currently scanning $h^{1,1} = 5, 6, 7, 8$
- Also looking at strong vs. weak Swiss Cheese detection
- CICYs for $h^{1,1} \leq 4$ are not Swiss Cheese, but may be for $h^{1,1} \geq 5$

  ➙ Still a lot to discover in the Swiss Cheese landscape...