G-structures and "moduli-mediated" no-scale SUSY breaking

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Primary references:

- Cámara, Graña: "No-scale supersymmetry breaking vacua and soft terms with torsion", hep-th/0710.4577
- Graña: "Flux compactification in string theory: a comprehensive review", hep-th/0509003

Motivation

In previous talks we discussed anomaly- and gravity-mediated SUSY-breaking in the context of the MSSM. The abundance of geometric moduli arising in string compactification scenarios provides a further method: ⇝ "moduli-mediated" SUSY-breaking

Spontaneous supersymmetry breaking occurs in the bulk due to F-terms of certain geometric moduli. This is mediated to D-branes, which are (partially) wrapping the internal geometry and thus are ultimately depending on the bulk moduli.

bulk sector "neutral matter"	∿∿∿ mediation by geometric moduli ∿∿∿	brane sector "charged matter"
spontaneous SUSY-breaking due to F-terms of the moduli		soft terms

Furthermore, the kind of SUSY breaking encountered here is of no-scale type, i.e. the cosmological constant is automatically vanishing (at least at tree level) and mass scales can be dynamically determined. However, this feature is not investigated any further within this talk.

I. SU(3)-structures and intrinsic torsion

G-structure = principal G-subbundle of the tangent bundle's associated frame bundle

⇝ allows to decompose tensors, n-forms, spinors, … in irreducible representations of G

⇝ see the book of Joyce "Compact manifolds with special holonomy" for mathematical details

- a connection on TM is compatible with the G-structure if its restriction to the principal G-subbundle is a valid connection as well

  ⇝ in general, a compatible connection will NOT coincide with the torsion-free Levi-Civita connection.

Why SU(3)-structures?

- internal manifold in string compactification has to be a 6d compact space (⇝ 6d compact Riemannian manifold)
- decomposition of the 6d spinor:

$$\underset{\substack{\text{(complex)}\\ SO(6)\text{-spinor}}}{4} \longrightarrow \underset{\substack{\text{fundamental}\\ SU(3)\text{-representation}}}{3} \oplus \underset{\substack{\text{trivial (!)}\\ \text{representation}}}{1}$$

⇝ SU(3)-invariant component

⇝ on manifolds admitting a SU(3)-structure, there exists a globally defined, non-vanishing SU(3)-invariant spinor

Description of SU(3)-structures

An SU(3)-structure can be described as follows:

ⓐ • a SU(3)-invariant spinor η
• a metric

ⓑ • a 2-form J
• a holomorphic 3-form Ω

such that $J \wedge \Omega = 0$

$$J_{mn} = \mp 2i\, \eta_\pm^\dagger \gamma_{mn} \eta_\pm$$

$$\Omega_{mnp} = -2i\, \eta_-^\dagger \gamma_{mnp} \eta_+$$

$$J \wedge J \wedge J = -i \frac{3}{4} \frac{N_J}{N_\Omega}\, \Omega \wedge \bar{\Omega}$$

where

$$\underbrace{N_J = \frac{1}{6} \int J \wedge J \wedge J \qquad N_\Omega = \frac{1}{8i} \int \Omega \wedge \bar{\Omega}}_{\text{normalization constants}}$$

- Ω induces an $\underbrace{\text{almost-complex structure}}_{\text{mapping } I: TM \to TM \text{ that squares to } -\mathrm{Id}}$ $I^m{}_n$

- J provides a $\underbrace{\text{symplectic structure}}_{\text{skew-symmetric mapping } TM \times TM \to \mathbb{R}}$ J_{mn}

- Ω and J define a metric: $\quad g_{mn} = J_{mp} I^p{}_n$
- if these forms are closed (i.e. $dJ=0$, $d\Omega$), the corresponding structures are integrable

Torsion classes of $SU(3)$-structures

- given an $SU(3)$-structure, there is always a compatible connection such that $\nabla_m g_{np} = 0$, $\nabla_m \eta = 0$

$\rightsquigarrow$ connection is in general not torsion-free

- for $SU(3)$-manifolds the torsion tensor $T_{mn}{}^p$ is a section of $\Lambda^1 \otimes (su(3) \oplus su(3)^\perp)$.
- acting on $SU(3)$-invariant forms, the $su(3)$-part drops

$\Rightarrow$ intrinsic torsion $T^i_{mn}\eta \in \Gamma(\Lambda^1 \otimes su(3)^\perp)$

- decomposition of the intrinsic torsion in irreducible $SU(3)$-representations

$$\underbrace{(3 \oplus \bar{3})}_{\text{real 1-form}} \otimes \underbrace{(1 \oplus 3 \oplus \bar{3})}_{\substack{\text{complement} \\ \text{to 8 in } so(6)}} = \underbrace{(1 \oplus 1)}_{W_1} \oplus \underbrace{(8 \oplus 8)}_{W_2} \oplus \underbrace{(6 \oplus \bar{6})}_{W_3} \oplus \underbrace{2(3 \oplus \bar{3})}_{W_4,\, W_5}$$

where
W_1 complex scalar
W_2 complex primitive $(1,1)$-form
W_3 real primitive $(2,1)+(1,2)$-form
W_4 real vector
W_5 complex $(1,0)$-form ($\approx$ real vector)

} 5 torsion classes of $SU(3)$-structures

- torsion classes allow to refine the $SU(3)$-structures non-integrability.

Integrability of SU(3)-structure

$$dJ = \frac{3}{2} \frac{\mathcal{N}_J}{\mathcal{N}_\Omega} \operatorname{Im}(\mathcal{W}_1 \bar{\Omega}) + \mathcal{W}_4 \wedge J + \mathcal{W}_3$$

$$d\Omega = \mathcal{W}_1 J + \mathcal{W}_2 \wedge J + \bar{\mathcal{W}}_5 \wedge \Omega$$

$\rightsquigarrow$ the vanishing of all 5 torsion classes implies that the SU(3)-structure is integrable $\Leftrightarrow$ Calabi-Yau manifold

II. No-scale SUSY breaking: general idea

- a Kähler potential K and a superpotential W define a scalar potential

$$V = e^K \sum_{i,j} \Big[(D_i W) K^{i\bar{j}} (D_{\bar{j}} \bar{W}) \overbrace{- 3 |W|^2}^{\text{negative piece}} \Big]$$

Kähler-covariant derivative $D_i W = \partial_i W + W \partial_i K$

inverse of the Kähler metric $K_{i\bar{j}} = \dfrac{\partial^2 K}{\partial \phi_i \partial \phi_j^*}$

running over all $\mathcal{N}=1$ moduli (more generally over all fields involved)

Choose a subset $\{\tilde{\imath}\}$ of moduli, such that

① the "no-scale condition" $\sum_{\tilde{\imath}} (\partial_{\tilde{\imath}} K) K^{\tilde{\imath}\bar{\tilde{\jmath}}} (\partial_{\bar{\tilde{\jmath}}} K) = 3$ is satisfied

② there are no mixed terms $K^{\tilde{\imath}\bar{j}}$ in the inverse of the Kähler metric

③ the superpotential is independent of the moduli $\{\tilde{\imath}\}$

$\Rightarrow$ the negative piece of the scalar potential is cancelled

$$\rightsquigarrow \quad V = e^K \sum_{i,j \neq \tilde{\imath}} (D_i W) K^{i\bar{j}} (D_{\bar{j}} \bar{W}) \quad \text{positive definite}$$

$\Rightarrow$ potential has an absolute minimum at $V=0$ when $D_i W = 0$ for all $i \neq \tilde{\imath}$

$$\boxed{\text{Since } D_{\tilde{\imath}} W \overset{③}{=} K_{\tilde{\imath}} W \neq 0, \text{ the supersymmetry is spontaneously broken at this minimum by the F-terms of the selected moduli}}$$

III. Type IIB compactification with O5/O9-planes

The previously outlined "moduli-mediated" SUSY-breaking will be considered in the context of type-IIB-orientifold compactification on SU(3)-manifolds with O5/O9-planes.

$\rightsquigarrow$ scalar geometric moduli arising from Kaluza-Klein mechanism

Basic setup:

- 4d Minkowski-flat warped spacetime

$$ds^2 = \underbrace{e^{2A(y)}}_{\text{warp factor}} \eta_{\mu\nu}\, dx^\mu dx^\nu + \underbrace{g_{mn}\, dx^m dx^n}_{\text{internal SU(3)-manifold}}$$

- in the presence of fluxes sources of negative tension / charge are required

  $\rightsquigarrow$ e.g. O5/O9-orientifold planes in type-IIB

- the orientifold projection also breaks down $\mathcal{N}=2$ SUSY of IIB to $\mathcal{N}=1$
- orientifold projection: worldsheet parity Ω_P + involution σ

$$\Rightarrow \quad \mathcal{O} = \Omega_P\, \sigma^*$$

involution for IIB with O5/O9: $\quad \sigma^* J = J$

$$\sigma^* \Omega = \Omega \quad \Rightarrow \quad \sigma^* I = I \qquad \begin{pmatrix}\text{holomorphic}\\ \text{involution}\end{pmatrix}$$

- RR field strengths:

$$F^{(10d)} = F + dvol_4 \wedge \lambda(*F) \qquad \text{for} \quad F = F_1 + F_3 + F_5 \quad \text{in IIB}$$

$$\text{where} \quad F_n = dC_{n-1} - H \wedge C_{n-3} + e^B \bar{F}$$

- bulk Kähler potential and superpotential

$$K = \underbrace{-\log\left(8\mathcal{N}_\Omega\right)}_{K_\Omega} \underbrace{-\log\left(8\, e^{-3\phi}\mathcal{N}_J\right)}_{K_J} \underbrace{-\log\left(S+S^*\right)}_{K_S}$$

$$W = \int \Omega \wedge G = \int \Omega \wedge \underbrace{\left(F_3 + i\, e^{-\phi}\, dJ\right)}_{\text{complex 3-flux}}$$

Where are the moduli?

$\rightsquigarrow$ the orientifold projection $\mathcal{O} = \Omega_P \sigma^*$ splits the $\mathcal{N}=2$ multiplets and all moduli are found as scalars of the resulting $\mathcal{N}=1$ chiral multiplets

multiplet	multiplicity	bosonic field content
gravity	1	$(g_{\mu\nu}, V_1^0)$
vector	$h^{(2,1)}$	(V_1^k, z^k)
hyper	$h^{(1,1)}$	(v^a, b^a, c^a, ρ_a)
tensor	1	(B_2, C_2, ϕ, C_0)

moduli of Ω $\rightsquigarrow$ coordinates of $\mathcal{M}_\Omega$

(complexified) moduli of J $\rightsquigarrow$ coordinates of $\mathcal{M}_J$

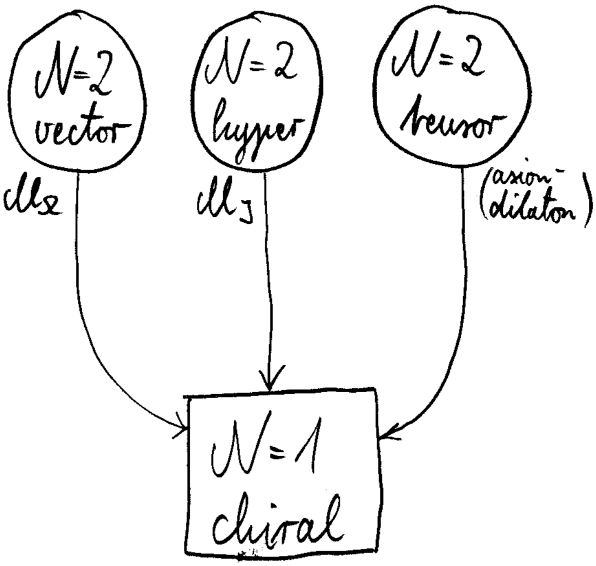
axion-dilaton modulus

4d $\mathcal{N}=2$ multiplets of type-IIB compactified on a SU(3)-manifold

$\Downarrow$ orientifold projection

multiplet	multiplicity	bosonic field content
gravity	1	$g_{\mu\nu}$
vector	$h^{(2,1)}$	V_1^k
chiral	$h_+^{(2,1)}$	z^k
	$h_+^{(1,1)}$	(v^α, c^α)
	$h_-^{(1,1)}$	(b^α, ρ_α)
	1	(C_2, ϕ)

4d $\mathcal{N}=1$ multiplets of IIB O5/O9



$\Rightarrow$ the moduli space consists of

① complex structure moduli U^k

② complexified Kähler deformations T^a

③ the axion-dilaton modulus $S = e^{-\phi}\mathcal{N}_J + iC_2$

IV. $\mathcal{N}=1$ vacua and SUSY-breaking

The 3-form flux $G = F_3 + i e^{-\phi} dJ$ can be decomposed as follows

1. $G = G^+ + G^-$ where $\underbrace{*_6 G^{\pm} = \pm i G^{\pm}}_{\text{imaginary (anti-) selfdual}}$

2. in terms of irreducible $SU(3)$-representations

$$G^+ = \frac{3}{2}\frac{\mathcal{N}_J}{\mathcal{N}_\Omega} G^+_{(1)} \bar{\Omega} + \cancel{G^+_{(3)} \wedge J} + G^+_{(6)}$$

$$G^- = \frac{3}{2}\frac{\mathcal{N}_J}{\mathcal{N}_\Omega} G^-_{(1)} \Omega + \cancel{G^-_{(3)} \wedge J} + G^-_{(6)}$$

vanish due to topological reasons

- the F-terms coming from the bulk superpotential are proportional to the Kähler-covariant derivative:

① complex moduli: $D_{U^k} W = -\int \chi_k \wedge G^-_{(6)}$

② Kähler moduli: $D_{T^a} W = 12 i e^{\phi} \left(\partial_{T^a} G^+_{(1)}\right) \mathcal{N}_J$

③ axion-dilaton modulus: $D_S W = (\partial_S K) W$

- if a purely imaginary self-dual (ISD-) flux $G = G^+$ is assumed, the complex moduli F-terms vanish

⇝ allows to express the superpotential as

$$W = 2 i e^{-\phi} \int \Omega \wedge dJ = -2 i e^{-\phi} \int d\Omega \wedge J = 12 i \mathcal{N}_J G^+_{(1)}$$

which allows to identify with the torsion classes

⇒ the vanishing of the F-terms ② and ③ can then be expressed as

$$\mathcal{W}_1 = F_{(1)} = 0$$
$$\mathcal{W}_3 = -e^{\phi} *_6 F_{(6)}$$

⇝ leads to a $\mathcal{N}=1$ supersymmetric vacuum.

Relaxing the conditions for SUSY-breaking

- let the internal $SU(3)$-manifold be a 2-cycle fibration

$$\underset{\substack{\text{2-cycle}\\\text{(complex 1-mfld)}}}{\Sigma_2} \hookrightarrow \underset{\substack{\text{internal}\\\text{manifold}}}{M_6} \twoheadrightarrow \underset{\text{4d base space}}{B_4}$$

$\rightsquigarrow$ splitting of the symplectic form $\quad J = J_B + J_{\Sigma_2}$

- assume:
  - the superpotential W does not depend on the Kähler moduli $T^{\tilde{a}}$ of the base
  - $G^+_{(1)}$ is non-vanishing, but independent of the Kähler moduli T^b of the fibre

$\rightsquigarrow$ corresponding ("relaxed") F-terms

① complex moduli: $\quad D_{U^k} W = - \int \chi_k \wedge \underset{\text{ISD-flux}}{G^-_{(6)}} = 0$

② Kähler moduli: $\quad D_{T^a} W = \begin{cases} (\partial_{T^{\tilde{a}}} K) W & \text{for } T^{\tilde{a}} \text{ (base)} \\ 0 & \text{for } T^b \text{ (fibre)} \end{cases}$

③ axion-dilaton modulus: $\quad D_S W = (\partial_S K) W$

$$\Downarrow$$

$$\left.\begin{aligned} \mathcal{W}_1 &= e^{\phi} F_{(1)} \\ \mathcal{W}_3 &= -e^{\phi} *_6 F_{(6)} \end{aligned}\right\} \text{corresponding relations between the torsion classes and the 3-flux}$$

$\rightsquigarrow$ if the sum in the scalar potential V runs over $i = T^b, U^k$, the scalar potential is positive definite

$\Rightarrow$ the supersymmetry breaking in the bulk is mediated through the F-terms of the moduli $T^{\tilde{a}}, S$.

V. D-branes and soft terms

The orientifold planes induce a global negative RR charge, which might not be completely cancelled by the flux.

$\Rightarrow$ need to add D5- and D9-branes, wrapping 2-cycles or the whole space, respectively.

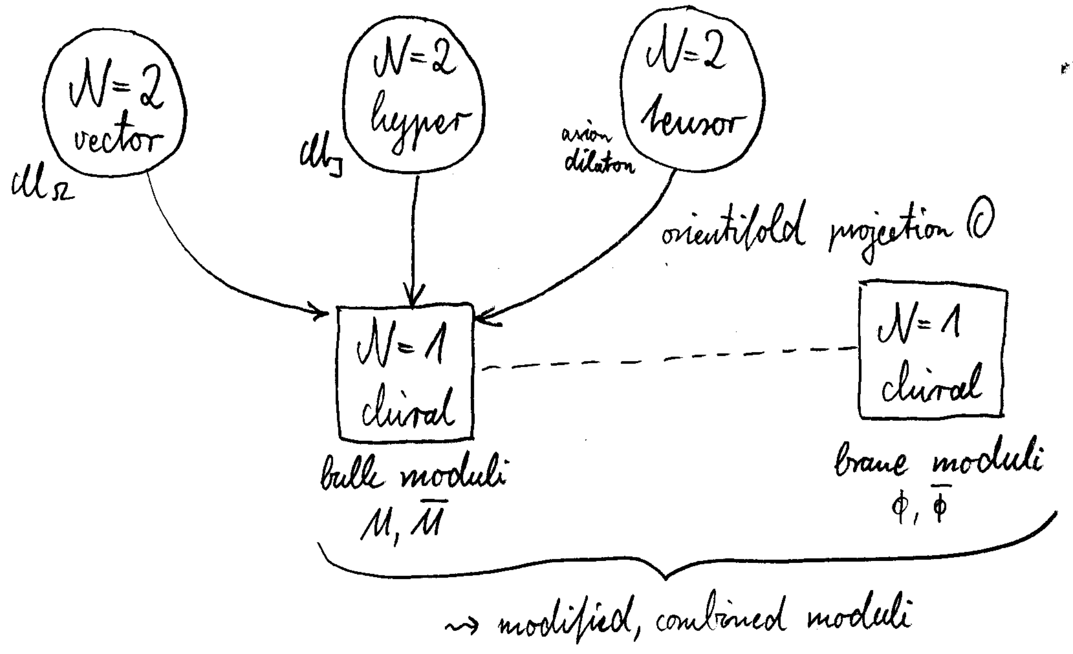
- on a SU(3)-manifold a supersymmetric D-brane is specified by a pair $(\Sigma, \mathcal{F})$

  $\mathcal{F}$: worldvolume flux $\mathcal{F} = F - P_\Sigma(B)$

  Σ: internal cycle which is wrapped

  and subject to a number of conditions (D-flatness, F-flatness, ...)
- deformations of the internal cycle are described by
  - holomorphic normal vectors
  - holomorphic 1-form gauge fields

  } brane moduli
- in the $\mathcal{N}=1$ SUGRA both the bulk and brane sector combine and the brane moduli form $\mathcal{N}=1$ chiral superfields ϕ^i, which couple to the geometric bulk moduli



$\Rightarrow$ brane fluctuations combine with the geometric bulk moduli

How are the brane moduli integrated?

If the VEV of the brane fluctuations vanishes, both the Kähler potential and the superpotential can be expanded in a power series of brane moduli:

$$K(M,\bar{M},\phi,\bar{\phi}) = \underbrace{\hat{K}(M,\bar{M})}_{\text{bulk Kähler potential}} + \underbrace{Z_{i\bar{j}}(M,\bar{M})\,\phi^i\bar{\phi}^{\bar{j}} + \frac{1}{2}\left[H_{ij}(M,\bar{M})\,\phi^i\phi^j + h.c.\right] + \ldots}_{K_{D_p}(M,\bar{M},\phi,\bar{\phi})}$$

$$W(M,\phi) = \underbrace{\hat{W}(M)}_{\text{bulk superpotential}} + \underbrace{\frac{1}{2}\tilde{\mu}_{ij}(M)\,\phi^i\phi^j + \frac{1}{6}\tilde{Y}_{ijk}(M)\,\phi^i\phi^j\phi^k + \ldots}_{W_{D_p}(M,\phi)}$$

⇝ inserting those expansions in the definition of the scalar potential V, one obtains an effective potential for the brane fields

$$V^{(eff)} = \underbrace{\left(\partial_i W^{(eff)}\right) Z^{i\bar{j}} \left(\partial_{\bar{j}} \bar{W}^{(eff)}\right)}_{\text{effective superpotential}} + \underbrace{m^2_{i\bar{j},soft}\,\phi^i\bar{\phi}^{\bar{j}} + \frac{1}{6}A_{ijk}\,\phi^i\phi^j\phi^k + \frac{1}{2}B_{ij}\,\phi^i\phi^j + h.c.}_{\text{soft SUSY breaking terms}}$$

where $W^{(eff)} = \frac{1}{2}\mu_{ij}\,\phi^i\phi^j + \frac{1}{3}Y_{ijk}\,\phi^i\phi^j\phi^k$

$$Y_{ijk} = e^{\frac{\hat{K}}{2M_{Pl}^2}}\,\tilde{Y}_{ijk}$$

$$\mu_{ij} = e^{\frac{\hat{K}}{2M_{Pl}^2}}\,\tilde{\mu}_{ij} + m_{\frac{3}{2}} H_{ij} - \bar{F}^{\bar{I}}\,\bar{\partial}_{\bar{I}} H_{ij}$$

gives a "supersymmetric" mass term $\Rightarrow m^2_{i\bar{j},susy}$

and the coefficients of the soft SUSY breaking terms are

$$m^2_{i\bar{j},soft} = |m_{\frac{3}{2}}|^2 Z_{i\bar{j}} - F^I \bar{F}^{\bar{J}} R_{I\bar{J}i\bar{j}}$$

$$A_{ijk} = F^I D_I Y_{ijk}$$

$$B_{ij} = 2|m_{\frac{3}{2}}|^2 H_{ij} - \bar{m}_{\frac{3}{2}} \bar{F}^{\bar{J}} \bar{\partial}_{\bar{J}} H_{ij} + m_{\frac{3}{2}} F^I D_I H_{ij}$$
$$- F^I \bar{F}^{\bar{J}} D_I \bar{\partial}_{\bar{J}} H_{ij} - e^{\frac{K}{2M_{Pl}^2}}\,\tilde{\mu}_{ij}\,\bar{m}_{\frac{3}{2}} + e^{\frac{K}{2M_{Pl}^2}} F^I D_I \tilde{\mu}_{ij}$$

gravitino mass: $m_{\frac{3}{2}} = e^{\frac{\hat{K}}{2M_{Pl}^2}}\,\frac{\hat{W}}{M_{Pl}^2}$ with M_{Pl} being the 4d Planck mass

Comments:

- additional mass terms and trilinear couplings are generated by the Giudice – Masiero mechanism (compare Martin's talk)

  $\Rightarrow$ total scalar masses for the brane fields

$$m^2_{i\bar{j}} = m^2_{i\bar{j},\,\text{susy}} + m^2_{i\bar{j},\,\text{GM}} + m^2_{i\bar{j},\,\text{soft}}$$

- note that due to $\hat{W} \propto G^+_{(1)}$ and $G^+_{(1)} = e^{-\phi} \mathcal{W}_1$ for ISD fluxes the gravitino mass

$$m_{\frac{3}{2}} = e^{\frac{\hat{K}}{2 M_{Pl}^2}} \frac{\hat{W}}{M_{Pl}^2} \propto \mathcal{W}_1$$

  is directly proportional to the first torsion class. A vanishing gravitino mass corresponds to the $\mathcal{N}=1$ conditions obtained in §IV.

  $\rightsquigarrow$ $\mathcal{W}_1$ allows to adjust the gravitino mass

- the model can be made completely explicit on suitable simple internal SU(3)-manifolds, e.g. twisted tori.

  $\rightsquigarrow$ see the original paper by Cámara & Graña

- an analogous model is presented for type-IIA with O6-planes