B. Jurke
21.1.2009

F-theory GUT group breaking via $U(1)_Y$-hypercharge fluxes

An introduction to the work of Donagi and Wijnholt

Primary references:

- Donagi, Wijnholt: "Model building with F-theory" arXiv: 0802.2969 [hep-th]
- Donagi, Wijnholt: "Breaking GUT groups in F-theory" arXiv: 0808.2223 [hep-th]

Motivation:

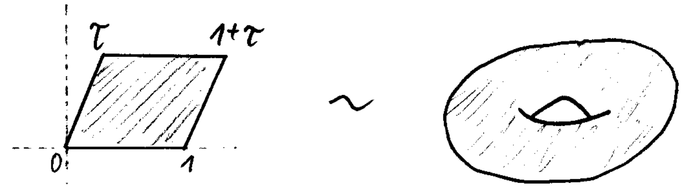
The treatment of S-selfdual type-IIB superstring theory in terms of F-theory provides a number of important ingredients, which are not found in the perturbative regime (or very hard to come by). One easily obtains the full range of ADE groups – in particular the exceptional ones – for the worldvolume gauge theory of 7-branes. The perturbatively inaccessible SO(10)-spinor representation 16 and Yukawa couplings descent from singularity / gauge group enhancements at the double- and triple intersection points of 7-branes.

In order to make contact with the low-energy MSSM, the GUT group must be broken, which is achieved by turning on an Abelian U(1)-hypercharge flux on the GUT 7-brane. Naturally, this has implications for the proton decay issue encountered in any GUT model.

I. Elements of "top-down" F-theory model building

The RR 0-form C_0 and the dilaton / string coupling are combined into the complex axion-dilaton scalar

$$\tau = C_0 + i e^{-\phi} \quad \Rightarrow$$



which parameterizes the complex structure of a 2d torus over the 10d type-IIB space-time.

Geometry: elliptically-fibered CY 4-fold with a Kähler 3-fold base (and a section σ that specifies the embedding of $B_3 \subset Y_4$)

$$\begin{array}{ccc} E & \hookrightarrow & Y_4 \\ & & \downarrow \\ & & B_3 \end{array}$$

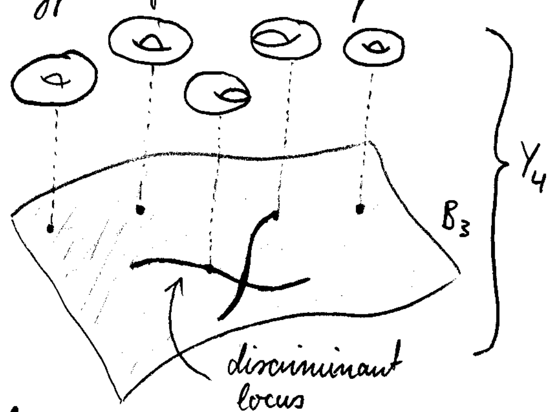
Locally this can be represented in Weierstrass form:

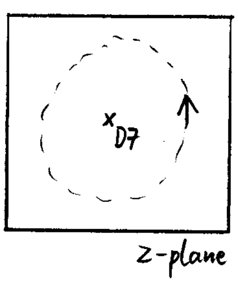
$$y^2 = x^3 + f x + g \qquad \text{where} \quad \begin{array}{l} f \in \mathcal{C}^\infty(K_{B_3}^{-4}) \\ g \in \mathcal{C}^\infty(K_{B_3}^{-6}) \end{array}$$

The complex structure of the elliptic fiber is encoded via

$$j(\tau) = \frac{4\,(24 f)^3}{\Delta} \qquad \text{with discriminant} \qquad \Delta = 4 f^3 + 27 g^2$$

7-branes: are located on the discriminant locus $\{\Delta = 0\} \subset B_3$ where the T^2-fiber degenerates. The type of the degeneration (→ Enriques-Kodaira classification of elliptic fibrations) determines the worldvolume gauge group on the 7-brane.





In the 2d space normal to the 7-brane worldvolume the axio-dilaton has a singularity

$$\tau(z) \sim \frac{1}{2\pi i} \log(z) \quad \Rightarrow \text{monodromy}$$

(→ Paul's talk, p. 4)

II. Abelian gauge fields in F-theory

An isolated 7-brane contains an $U(1)$ gauge field A_μ on its worldvolume. The ultimate goal is to turn on a background flux on the worldvolume.

In F-theory there is only the G_4 flux, which encodes both the NS- and RR-3-form flux as well as the worldvolume fluxes. To extract the worldvolume flux, we use the M-theory origin of F-theory:

F-theory on $\mathbb{R}^{1,2} \times S^1_R \times Y_4$	$\sim$	M-theory on $\mathbb{R}^{1,2} \times Y_4$ (where the area (volume) of the elliptic fibre is proportional to $\frac{1}{R}$)

In M-theory gauge fields arise from expanding the 3-form potential $\hat{C}_3$ along harmonic 2-forms:

$$\hat{C}_3 = A \wedge \omega, \qquad \rightsquigarrow \text{ try the same in F-theory}$$

which couples naturally to M2-branes.

Ⓐ "pure base" case: M2-brane wrapping a 2-cycle α_2 of B_3

$\Updownarrow$ F-/M-theory duality

D3-brane wrapping $S^1_R \times \alpha_2$

$\downarrow$ decompactification limit $(R \to \infty)$

$\Rightarrow$ string in 4 non-compact dimensions (which couples to a pseudoscalar)

$\lightning$: no 4d vector field

Ⓑ "pure fibre" case: M2-brane wrapping the fibre

$\Updownarrow$ F-/M-theory duality

fundamental string with momentum along S^1_R

$\downarrow$ decompactification limit

$\Rightarrow$ component of the 4d metric

$\lightning$: no 4d vector field

Ⓒ mixed case: M2-brane wrapping a remaining cycle of Y_4

$\Updownarrow$ F-/M-theory duality

(p,q) - strings

open (p,q)-string couples electrically to 7-branes

closed (p,q)-string couples to a linear combination of bulk NS and RR 2-forms with one index on the base

$\Rightarrow$ gauge fields in 4 non-compact dimensions

Therefore one obtains a lattice of harmonic 2-forms:

$$\Lambda = \{ \omega \in H^2(Y_4) \;:\; \omega \cdot \alpha = 0 \quad \text{for} \quad \underbrace{\alpha \in H_2(B_3)}_{\text{"pure base"}} \quad \text{or} \quad \underbrace{\alpha = [T^2]}_{\text{"pure fibre"}} \}$$

The harmonic 2-form also serves as a kind of δ-function that peaks on the 7-brane world-volume S, such that

$$\frac{C_3}{2\pi} = A_S \wedge \omega^S$$

gives the $U(1)$ gauge field A_S on S. The G_4-flux

$$G_4 = F_S \wedge \omega^S$$

then describes the corresponding flux along the 7-brane.

$$\boxed{\begin{array}{l}\text{Recall: harmonic forms} \\ \mathcal{H}^k_\Delta(M) = \{ \alpha \in \Omega^k(M) : \Delta\alpha = 0 \} \\ \text{Hodge theorem: } \mathcal{H}^k_\Delta(M) \cong H^k(M)\end{array}}$$

III. Consistency conditions for the G-flux

The G_4-flux is subject to a number of constraints and conditions:

① requirement of a stable Minkowski vacuum:

$$G \in \Omega^{2,2} \qquad (\text{instead of } G \in \Omega^{2,2} \oplus \Omega^{4,0} \oplus \Omega^{0,4})$$

② D-term: $\underset{\substack{\uparrow \\ \text{Kähler form on the base } B_3}}{J} \wedge G = 0 \quad \rightsquigarrow \quad G$ primitive w.r.t. J

③ tadpole cancellation condition:

$$N_{D3} = \frac{\chi(Y_4)}{24} - \frac{1}{8\pi^2}\int_{Y_4} G \wedge G$$

④ quantization condition:

$$\left[\frac{G}{2\pi}\right] - \frac{p_1(Y_4)}{4} \in H^4(Y_4; \mathbb{Z})$$

(This rather innocent looking quantization condition actually leads to a number of subtleties related to the Freed-Witten anomaly. In the Donagi-Wijnholt papers those problems are "covered" by splitting the spacetime pullback of the worldvolume gauge bundle:

$$i^*A = A_S - \frac{1}{2}A_N$$
)

IV. Breaking the GUT group

In general, there are three possible approaches for breaking the GUT group in string models:

① Engineer an <u>adjoint Higgs</u> or Higgs in another large representation

<u>problem</u>: One ends up with traditional 4d GUT models and all of their respective problems (doublet-triplet splitting, proton decay vs. unification issue)

② Using the heterotic / F-theory duality, one may consider <u>discrete Wilson lines</u>.

<u>problem</u>: In order for the decoupling principle to work and to avoid adjoint Higgses, the GUT 7-brane needs to wrap a del Pezzo surface S $\Rightarrow$ $\pi_1(S) = 0$

So there are no 1-cycles for Wilson lines

⇒ ③ Turn on <u>$U(1)$-hypercharge fluxes</u> on the 7-brane.

SU(5)-GUT example:

Let the GUT 7-brane with gauge group $G = SU(5)$ wrap the 4-cycle $S \subset B_3$. Furthermore, define an $\mathfrak{su}(5)$-matrix

$$\Upsilon = \begin{pmatrix} -\frac{1}{3} & & & & \\ & -\frac{1}{3} & & 0 & \\ & & -\frac{1}{3} & & \\ & 0 & & \frac{1}{2} & \\ & & & & \frac{1}{2} \end{pmatrix} \Rightarrow \underbrace{\underset{\text{traceless}}{\mathrm{Tr}\,\Upsilon = 0}, \quad \underset{\text{Hermitean}}{\Upsilon^\dagger = \Upsilon}}_{\text{generator of } SU(5)}$$

which specifies the embedding of the Abelian hyperflux into the GUT group.

Basic idea: For the $U(1)$-hypercharge let $L \overset{\pi}{\twoheadrightarrow} S$ be a (complex) line bundle, which is entirely specified by a harmonic 2-form $c_1(L) \in H^2(S) \cong \mathcal{H}^2_\Delta(S)$ induced by the first Chern class, and set the $SU(5)$ field strength to

$$\langle F_{SU(5)} \rangle = c_1(L)\, \Upsilon$$

embedding matrix

Chern-Weil homomorphism: $c_1(L) = \left[-\frac{1}{2\pi i} F_L\right]$

Realization in F-theory: Let ω^Y denote the corresponding harmonic 2-form for the hypercharge flux localized on the GUT 7-brane. The $U(1)$-hypercharge gauge field is found in

$$C_3 = A_Y \wedge \omega^Y$$

(using the M-theory origin)

Turning on the above hypercharge flux corresponds to

$$\frac{G_4}{2\pi} = c_1(L) \wedge \omega^Y$$

$\Rightarrow$ Breaks the gauge group on the GUT 7-brane to the commutant of $U(1) \subset SU(5)$ using the embedding Υ

$$SU(5) \xrightarrow{U(1)_Y\text{-flux embedded via } \Upsilon} SU(3) \times SU(2) \times U(1)$$

Problems with the naive approach:

- Turning on this particular $U(1)$-flux leads to unacceptable triplets in the spectrum $\rightsquigarrow$ consider a slightly generalized flux
- Due to the coupling of the worldvolume gauge theory to the bulk, the Abelian gauge fields may pick up a mass

$$\Pi^Y_M = -2\pi \, \mathrm{Tr}(T^2) \int_S c_1(L^Y) \wedge i^* \beta_M \longleftarrow \text{basis of } H^2(B_3)$$

through the Stückelberg mechanism

$$\Downarrow$$

Partial solution: Turn on a flux in the same cohomology class $[c_1(L^Y)] \in H^2(S)$ in a hidden sector.

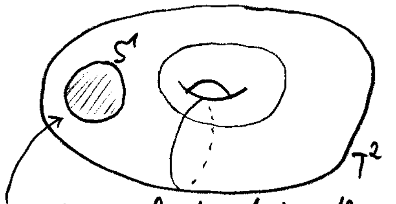
BUT: One looses the standard $SU(5)$-GUT relations

$$\sqrt{\tfrac{5}{3}}\, g_Y = g_{SU(2)} = g_{SU(3)}$$

Ultimately, one aims to get $\Pi^Y_M = 0$, i.e. the $U(1)$-hypercharge gauge bosons should remain massless.

Key observation: Let $\tilde{c}_1(L^Y) \in H_2(S)$ denote the Poincaré-dual to $c_1(L^Y) \in H^2(S)$. The embedding (inclusion) $i: S \hookrightarrow B_3$ induces the homological inclusion $i_*: H_2(S) \longrightarrow H_2(B_3)$.
Then it follows:

$$\boxed{\Pi^Y_M = 0 \quad \text{for all } M \quad \Longleftrightarrow \quad \tilde{c}_1(L^Y) \in \ker(i_*)}$$



a circle embedded in this fashion becomes a trivial 1-cycle of T^2, but on its own it represents a non-trivial 1-cycle

i.e. while $\tilde{c}_1(L^Y)$ may be a nontrivial 2-cycle in S, once embedded in B_3 it becomes trivial (that is a boundary).

Remark: In cohomology the above statement is equivalent to requiring $c_1(L)$ to be a generator of $\mathrm{coker}(i^*)$.

V. Fractional bundles and generalized $U(1)_Y$-flux

Two intersecting 7-branes give rise to chiral matter due to the decomposition of the enhanced singularity / gauge group $G_\Sigma \supset G_S \times G_{S'}$ at the intersection: (→ recall Paul's talk, p. 8)

$$\mathrm{ad}(G_\Sigma) = \mathrm{ad}(G_S) \oplus \mathrm{ad}(G_{S'}) \oplus \bigoplus_j U_j \otimes U_{j'}$$

The fermions on the intersections also couple to the gauge bundles. Using the identification $S_\Sigma^{+} = K_\Sigma^{\frac{1}{2}}$ and after introducing "fake" gauge bundles $\tilde{E} = E \otimes N_S^{-\frac{1}{2}}$ and $\tilde{E}' = E' \otimes N_{S'}^{-\frac{1}{2}}$, the zero modes of the Dirac operator correspond to the generators of the cohomology group

$$H^i(\Sigma; \mathcal{G}) \qquad \text{where} \qquad \mathcal{G} = \left[U_j(\tilde{E}) \otimes U_j'(\tilde{E}')\right]\Big|_\Sigma \otimes K_\Sigma^{\frac{1}{2}}.$$

The degree i corresponds to the 4d chirality ("even/odd").

<u>Phenomenological constraints:</u> Consider the decomposition of certain $SU(5)$-representations under $SU(3)\times SU(2)\times U(1)_Y$:

$$\begin{array}{lll}
\text{adjoint:} & 24 \mapsto & (8,1)_0 \oplus (1,3)_0 \oplus (1,1)_0 \oplus (3,2)_{-\frac{5}{6}} \oplus \underbrace{(\overline{3},2)_{\frac{5}{6}}}_{\text{unwanted triplet}} \\
\text{2-form:} & 10 \mapsto & (3,2)_{\frac{1}{6}} \oplus (\overline{3},1)_{-\frac{2}{3}} \oplus (1,1)_1 \\
\text{conjugate:} & \overline{5} \mapsto & (\overline{3},1)_{\frac{1}{3}} \oplus (1,2)_{-\frac{1}{2}}
\end{array}$$

In order to get the MSSM at low energies, the unwanted triplet has to vanish.

$$\Rightarrow \; 0 = \chi(S, L^{\frac{5}{6}}) = 1 + \tfrac{1}{2} c_1(L^{\frac{5}{6}})^2 \quad \Leftrightarrow \quad c_1(L^{\frac{5}{6}})^2 = -2$$

Furthermore, $c_1(L^{\frac{1}{6}})$ and $c_1(L^{\frac{1}{3}})$ need to be integer classes for the other representations to make sense. Note that this also implies that $L^{\frac{5}{6}}, L^{\frac{1}{6}}, L^{\frac{1}{3}} \twoheadrightarrow S$ have to exist as honest bundles on S. However, this is incompatible with the condition $c_1(L^{\frac{5}{6}})^2 = -2$!

⇝ need to turn on a more general $U(1)_Y$-flux

Basic idea: Localize the bundle conditions! Add the appropriate 7-branes, such that

$$\begin{array}{ll} 10 \text{ loc. on } \Sigma_{10} & M_{10} \to \Sigma_{10} \\ \bar{5} \text{ loc. on } \Sigma_{\bar{5}}\,, & M_{\bar{5}} \to \Sigma_{\bar{5}} \end{array}$$

Line bundles which arise from turning on a G-flux to yield 3 unbroken SU(5)-generations

If the hypercharge flux on S is turned on, the relevant massless spectrum is localized on the matter curves, i.e.

$$H^i\left(\Sigma_{10}\,;\, L^{\frac{1}{6}}\big|_{\Sigma_{10}} \otimes M_{10} \otimes K^{\frac{1}{2}}_{\Sigma_{10}}\right)$$

$$H^i\left(\Sigma_{\bar{5}}\,;\, L^{\frac{1}{3}}\big|_{\Sigma_{\bar{5}}} \otimes M_{\bar{5}} \otimes K^{\frac{1}{2}}_{\Sigma_{\bar{5}}}\right)$$

Therefore, the conditions are reduced to the bundles

$$L^{\frac{5}{6}} \to S \qquad \text{(as before)}$$

$$\left.\begin{array}{l} L^{\frac{1}{6}}\big|_{\Sigma_{10}} \otimes M_{10} \otimes K^{\frac{1}{2}}_{\Sigma_{10}} \to \Sigma_{10} \\ L^{\frac{1}{3}}\big|_{\Sigma_{\bar{5}}} \otimes M_{\bar{5}} \otimes K^{\frac{1}{2}}_{\Sigma_{\bar{5}}} \to \Sigma_{\bar{5}} \end{array}\right\} \text{ subspaces of } S,$$

i.e. only one global and two localized conditions. Therefore, the bundles $L^{\frac{1}{6}}, L^{\frac{1}{3}} \to S$ do not need to exist by themselves.

A comment on heterotic duals: Using the heterotic/F-theory duality one might try to consider the corresponding counterpart. However,

F-theory models using the $U(1)_Y$-flux GUT breaking mechanism cannot have heterotic duals.

When making the heterotic/F-theory duality explicit, see Donagi-Wijnholt I, §3, one considers a fibration $B_3 \to S$ of the base over the GUT 7-brane. Thus the inclusion-induced cohomological restriction mapping

$$i^*: H^2(B_3) \longrightarrow H^2(S) \quad \text{is surjective,}$$

which means that any $c_1(L) \in H^2(S)$ can be pulled back to B_3. However, the surjectivity also implies

$$\mathrm{coker}\,(i^*) = H^2(S) / \underbrace{\mathrm{im}\,(i^*)}_{H^2(S)} = 0.$$

But in order to keep the hypercharge gauge bosons massless (protected from the Stückelberg mechanism, p. 7) it is required that $c_1(L) \in \mathrm{coker}(i^*)$.

Therefore, the discussed $U(1)_Y$-hyperflux mechanism for GUT group breaking cannot be applied to the heterotic side.

$$\Downarrow$$

$$\boxed{\text{Breaking the GUT group by hypercharge fluxes is intrinsic to F-theory (so far...)}}$$