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Evidence of F(uzz) Theory

Non-commutative aspects of the decoupling limit

Primary references:

- Heckman, Verlinde: "Evidence for F(uzz)-Theory" arXiv: 1005:3033 [hep-th]

Motivation:

The term F-theory was first used by Vafa in his 1994 paper as a theory that geometrically encodes the S-selfduality of the type-IIB superstring. However, it more or less vanished from the mainstream about 2 years later. Since 2008 F-theory has sort of a "second spring" due to a couple of discoveries:

- the "local model" approach in the F-theory framework that allows to "ignore" the challenges faced in global model building
- the natural existence of exceptional gauge groups, which are important for certain phenomenologically relevant Yukawa couplings - those are absent in perturbative IIB orientifold models.

The "local F-theory model" approach has been subject to many debates, most importantly with respect to the decoupling principle.

I. A brief summary of F-theory

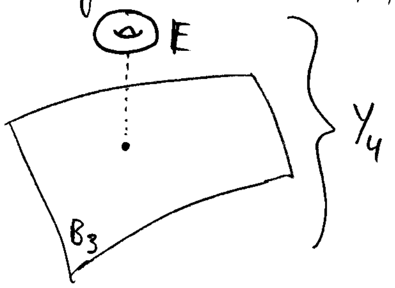
F-theory is usually viewed as the non-perturbative completion of IIB string theory. It can be defined via dualities from M-theory

F-theory on $\mathbb{R}^{1,2} \times S^1_R \times Y_4$	$\sim$	M-theory on $\mathbb{R}^{1,2} \times Y_4$ where the fibre volume is proportional to $\frac{1}{R}$
↓		↓
type-IIB superstring (10d)	← T-duality →	type-IIA superstring (10d)

However, this indirect definition is not very practical to work with.

Typical F-theory setting: $\mathbb{R}^{1,3} \times Y_4$, Y_4 elliptically-fibered CY 4-fold

$$\mathbb{E} \hookrightarrow Y_4 \to B_3$$



The elliptic fibre's complex structure is associated with the axio-dilaton $\tau = C_0 + i e^{-\Phi}$

$\Downarrow$

- description of the fibre: $y^2 = x^3 + f(z_i)\, x + g(z_i)$ — Weierstrass model — parameterized by f, g

  ($\mathbb{CP}^2_{2,3,1}[6]$ - hypersurface)

- elliptic discriminant: $\Delta = 4f^3 + 27g^2$

  discriminant locus $\Delta = 0 \iff$ degeneration of the fibre

  $\Downarrow$

  corresponds to the location of 7-branes

$$\tau(z) \sim \frac{1}{2\pi i} \log(z)$$

local structure of the axio-dilaton close to a 7-brane in transverse space at $z = 0$.

$$\Delta = \prod_\alpha \Delta_\alpha(z_i) = 0$$

$\Delta_\alpha(z_i)$: irreducible components → 7-brane

- singularity type: vanishing degrees of Δ, f and g determine the singularity type
  ⇝ Kodaira & Tate classification

Typical F-theory GUT setting: gauge group $SU(5)$ (or $SO(10)$)

coordinates are chosen suitably, such that the 7-brane carrying the GUT group takes the form

$$\{\Delta_{GUT} = 0\} = \{z = 0\} \quad \rightsquigarrow \quad \Delta = z^5 \prod_{\alpha \neq GUT} \Delta_\alpha(z, z_j)$$

Δ has vanishing degree 5 for an $SU(5)$ GUT group

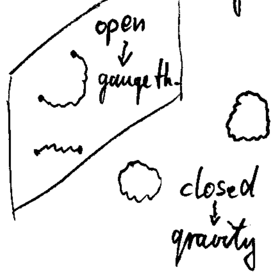
- from the local perspective of the GUT 7-brane only the intersections with other 7-branes are relevant

$$\underbrace{\Sigma_\alpha = \{\Delta_\alpha = 0\} \cap \{z = 0\}}_{\text{matter curve}} \quad \rightsquigarrow \text{6d localized matter}$$

- Yukawas and couplings arise from double/triple intersections of matter curves.

II. The decoupling principle

Local string model building: the relevant degrees of freedom for the 4d theory are localized in a small neighborhood of the internal manifold. ⇒ ignore global structure



⇝ allows for decoupling of the open & closed string sectors

The decoupling effectively sends off the 4d Planck scale

$M_{Pl}^{4d} \longrightarrow \infty$ ⇝ leaves 4d QFT

Approach in local F-theory model building:

⇒ GUT brane must wrap del Pezzo surface S

$dP_n, n=0,\dots,8$ is the n-fold blowup of $\mathbb{CP}^2$

Why? dP_n's are rigid ⇒ only have a volume modulus

⇝ allows to shrink/resize the dP_n without changing anything else.

In order to obtain the effective QFT of the local GUT setting we take in fact two limits:

$\mathrm{Vol}(S) \longrightarrow 0$ ⇝ pushes up the KK mass scale

$\alpha' \longrightarrow 0$ ⇝ decouples massive string excitations / corresponds to $T_s \to \infty$

⇝ massless point particle limit

(NOTE: in global settings there's the issue of stabilizing the GUT volume modulus dynamically)

Big question: Why does the "geometric perspective", i.e. the information contained in intersections etc., remain valid even in the zero volume limit?

⇝ Validity / well-definedness of the decoupling limit

Consider the D-brane Dirac-Born-Infeld (DBI) action:

$$S_{DBI} = - T_{D_p} \int_S d^{p+1}\xi \;\; e^{-\Phi} \sqrt{-\det\,(G_{ab} + B_{ab} + F_{ab})}$$

$F = dA$ U(1)-worldvolume flux

pullback of NS 2-form

(effective) "open string metric"

$\Rightarrow \;\; G_{open} = G_{closed} + \mathcal{B}$

the "closed string metric" is the naive geometrical metric G

combined background fields/fluxes $\mathcal{B} = B + F$

more generally: $\mathcal{B} = F + p\, B_{NS} + q\, B_{RR}$ (B_{RR}: C_2)

$\Rightarrow \;\; Vol_{open}(S) = Vol_{closed}(S) + \int_S \mathcal{B} \wedge \mathcal{B}$

Therefore, in the geometrical zero size limit: $\;\; V_{open}(S) \neq 0$

effective 4d gauge coupling:

$$\frac{4\pi}{g_{GUT}^2} = \frac{Vol_{open}(S)}{(2\pi)^2\, g_s} \xrightarrow{\text{zero size limit}} \frac{1}{(2\pi)^2\, g_s} \int_S \mathcal{B} \wedge \mathcal{B}$$

$\rightsquigarrow$ in the zero size limit the background fields/fluxes govern the gauge coupling (and dynamics)

in FLAT space: $\rightsquigarrow$ the limit we've considered here precisely corresponds to the Seiberg-Witten limit

(zero slope limit $\alpha' \to 0$, zero volume limit $Vol(S) \to 0$ | $Vol_{open}(S)$ finite, $\mathcal{B} = B + F$ non-zero)

$\Downarrow$ Seiberg, Witten (arXiv: hep-th/9908142) "String theory and non-commutative geometry"

non-commutative geometry along in internal directions

CLAIM in the paper: ⇝ analogous conclusion in the non-flat setting

⇒ suggests treating S as a non-commutative space S_{NC}

⇓

Fuzz theory

III. Non-commutative geometry primer

Basic idea: turn the space in a quantum system

a) Coordinates: replace coordinates z_i by bosonic harmonic oszillators

$$z_i \longrightarrow \hat{z}_i \qquad [\hat{z}_i, \hat{z}_j^{\dagger}] = \hbar_{NC}\, \delta_{ij}$$
$$\bar{z}_j \longrightarrow \hat{z}_j^{\dagger}$$

↑ (quantization parameter ⇝ determines size of "Planck cell")

The operators act on a Fock space of states:

$$\mathcal{F}(\mathbb{C}^r) = \mathrm{span}_{\mathbb{C}} \left\{ \prod_{i=1}^{r} \frac{(\hat{z}_i^{\dagger})^{n_i}}{\sqrt{n_i!}} |0\rangle, \quad n_i \in \mathbb{N} \right\}$$

↑ vacuum state ⇝ $\hat{z}_i|0\rangle = 0$

The basis elements of this Fock space correspond to the quantized points of $\mathbb{C}^r$

⇝ in general: # points in S_{NC} = $\dim_{\mathbb{C}} \mathcal{F}(S)$

b) differential forms: introduce further fermionic harmonic oszillators

$$dz_i \to \hat{C}_i \qquad \{\hat{C}_i, \hat{C}_j^\dagger\} = \hbar_{NC}\,\delta_{ij}$$

$$d\bar{z}_j \to \hat{C}_j^\dagger$$

note: this spoils $dz \wedge d\bar{z} = -d\bar{z} \wedge dz$!

which requires an expanded Fock space for the exterior algebra

$$\mathcal{F}(\Lambda^\bullet T^*_{hol}\mathbb{C}^r) = \mathrm{span}_{\mathbb{C}} \left\{ \prod_{i=1}^{r} \frac{(\hat{\bar{z}}_i^\dagger)^{n_i}}{\sqrt{n_i!}} (\hat{C}_i^\dagger)^{\sigma_i} |0\rangle \ : \ \begin{array}{l} n_i \in \mathbb{N} \\ \sigma_i \in \{0,1\} \end{array} \right\}$$

$\cup$

$\mathcal{F}(\mathbb{C}^r)$

new vacuum state: $\hat{C}_i|0\rangle = 0$, $\hat{\bar{z}}_j|0\rangle = 0$

The subspace of $(n,0)$-forms is then spanned by those elements where n of the σ_i's are 1.

c) Functions: Using $\mathcal{F}(\Lambda^\bullet T^*_{\overline{hol}}\mathbb{C}^r) \cong \mathcal{F}(\Lambda^\bullet T^*_{hol}\mathbb{C}^r)^*$ we observe that (p,q)-forms in general are linear mappings ("operators")

$$\mathcal{F}(\Lambda^\bullet T^*_{hol}\mathbb{C}^r) \longrightarrow \mathcal{F}(\Lambda^\bullet T^*_{hol}\mathbb{C}^r),$$

such that functions as $(0,0)$-forms are operators

$$\mathcal{F}(\mathbb{C}^r) \to \mathcal{F}(\mathbb{C}^r).$$

We write differential forms as

$$\omega_{(p,q)} = \sum_{I,J} f_{I,J}(\hat{\bar{z}}^\dagger, \hat{\bar{z}}) \underbrace{\hat{C}_{i_1} \wedge \cdots \wedge \hat{C}_{i_p}}_{I=(i_1,\dots,i_p)} \wedge \underbrace{\hat{C}_{j_1}^\dagger \wedge \cdots \wedge \hat{C}_{j_q}^\dagger}_{J=(j_1,\dots,j_q)}$$

wedge product $\hat{=}$ operator product

NOTE: The big difference to the commutative case is

(p,q)-form $\wedge$ (p',q')-form $=$ mixed form !

NOT $(p+p', q+q')$-form (in general)

d) Subspaces: let P be a function (polynomial). We define a "fuzzy differential":

$$dP = [\partial, P] \qquad \text{where} \qquad \partial = \sum_{i=1}^{r} \hat{c}_i \hat{z}_i^{\dagger}$$

Then the states representing the subspace $\{P=0\}$ are

$$\mathcal{F}_P(\mathbb{C}^r) = \ker(P) \cap \ker(dP)$$

$$= \left\{ |\psi\rangle \in \mathcal{F}(\mathbb{C}^r) : \begin{array}{l} P(\hat{z}_i^{\dagger}, \hat{z}) \, |\psi\rangle = 0 \\ dP \, |\psi\rangle = 0 \end{array} \right\}$$

"subspace Fock state space"

Let $\{|p\rangle\}$ be a basis for $\mathcal{F}_P$

$$\pi_P = \sum_{|p\rangle \in \mathcal{F}_P} |p\rangle\langle p| \qquad \text{projection operator onto } \mathcal{F}_P$$

Differential forms on the subspace defined by "sandwiching"

$$\omega^{P}_{(p,q)} = \pi_P \cdot \omega_{(p,q)} \cdot \pi_P$$

e) Compact spaces: X compact (toric) space

$\Rightarrow$ $\mathcal{F}(X)$ has finite dimension!

$\rightsquigarrow$ # points in X_{NC} = $\dim_{\mathbb{C}} \mathcal{F}(X)$ finite.

IV. Fuzz theory

Fuzz theory is the non-commutative "decoupling limit" theory where the GUT-brane divisor S is non-commutative

F-theory

$\Downarrow$

GUT 7-brane worldvolume theory $\mathbb{R}^{1,3} \times S$ $\longrightarrow$ Fuzz theory: "fuzzy 7-brane" on $\mathbb{R}^{1,3} \times S_{NC}$

A couple of observations:

- The fuzzy space S_{NC} consists of a finite number of points
  integration is replaced by a trace

$$\rightsquigarrow \quad L = \sum_{|p\rangle \in \mathcal{F}_S} \langle p | \mathcal{L}^{(0)}(\hat{z}^\dagger, \hat{z}) | p \rangle \qquad \text{effective fuzzy Lagrangian}$$

bosonic projection

$$\mathcal{L}^{(0)}(\hat{z}^\dagger, \hat{z}) = {}_c\langle 0 | \mathcal{L}(\hat{z}^\dagger, \hat{z}, \hat{c}, \hat{c}^\dagger) | 0 \rangle_c$$

$\Rightarrow$ the usually infinite sum of KK excitations effectively truncates after finite many degrees of freedom ($\rightsquigarrow$ regularization)

- The "continuum" of gauge couplings is turned into a "discretuum"

$$\frac{g_{YM}^2}{4\pi} = \frac{g_s}{N} \qquad \text{where} \quad N = \#\ \text{points in } S_{NC}$$

- 1-loop perturbation theory remains valid up to high energies (due to the truncated tower of dynamic KK modes)

Matter delocalization:

In the commutative theory, matter is localized on matter curves

$$\Sigma_\alpha = \{\Delta_{GUT} = 0\} \cap \{z = 0\}$$

$\rightsquigarrow$ well-defined matter curves in any case

In the non-commutative setting the canonical analogue to the subspace may result in an empty space due to the non-density of the (finite) S_{NC}.

$$\mathcal{F}_{\Sigma_\alpha} = \emptyset \quad \rightsquigarrow \quad \text{notion of "fuzzy matter curve" breaks down}$$

In this case the projection operator π_{Σ_α} degrades to the identity

⇒ matter field spreads out over the entire space

⇓

new sources for the generation of Yukawa couplings //

<u>mentionable:</u>

- fuzzy flop transitions
- fuzzy Landau-Ginzburg models
- holographic dual:

  7-brane on N-point fuzzy space ≅ tiled by N D3-branes

  → commutative limit ≅ dense mesh of D3's

  ⇓ ?

  similar: extremal black holes in IIA ~ D0-brane tiling of the horizon

  <u>suggests:</u> interpolation between

  single fuzzy SU(5)-GUT model on 7-brane

  ↕

  collective dynamics of $5 \cdot N$ D3-branes